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Chiral 4d string vacua with D-branes and NSNS and RR fluxes

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Abstract: We discuss type-IIB orientifolds with D-branes, and NSNS and RR field strength fluxes. The D-brane sectors lead to open string spectra with non-abelian gauge symmetry and charged chiral fermions. The closed string field strengths generate a scalar potential stabilizing most moduli. We describe the construction of models with $\mathcal{N} = 1$ supersymmetric subsectors in the context of orientifolds of IIB theory on $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, containing D9-branes with world-volume magnetic fluxes, and illustrate model building possibilities with several explicit examples. We comment on a T-dual picture with D8-branes on non-Calabi-Yau half-flat geometries, and discuss some of the topological properties of such configurations. We also explore the construction of models with fluxes and with D3-branes at singularities and present a non-supersymmetric 3-family SU(5) model.

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1. Introduction

One of the most remarkable features of string theory is that, despite its complexity, it admits vacua with low-enery physics surprisingly close to the structure of observed particles and interactions. In particular there exist by now several classes of setups (e.g. heterotic compactifications, type-II models with D-branes at singularities, or intersecting D-branes, compactifications of Horava-Witten theory, etc) leading to four-dimensional gravitational and non-abelian gauge interactions, with charged chiral fermions. Within each class, particular explicit models with spectrum very close to that of the (Minimal Supersymmetric) Standard Model have been constructed. On the other hand, a generic feature of all these constructions, is the existence of a (very often large) number of moduli, which remain massless in the construction, unless some supersymmetry breaking mechanism is proposed. From this viewpoint these models are relatively far from describing physics similar to the observed world.

Recently, it has been shown that, in the setup of Calabi-Yau compactification of type-II string theory (or also M-theory), there exists a natural mechanism which stabilizes most moduli of the compactification. This is achieved by considering compactifications with non-trivial field strength fluxes for closed string p-form fields. This proposal has already been explored in different setups, leading to large classes of models with very few unstabilized moduli. Hence this mechanism is one of the most interesting recent insights to address the long-standing problems of moduli in phenomenological string models. Unfortunately, compactifications with NSNS and RR field strength fluxes have centered on simple models, which lead to uninteresting gauge sectors, from the phenomenological viewpoint. In particular, the class of models studied are naturally non-chiral, since the corresponding gauge sectors arise from too simple stacks of parallel D-branes.

Our purpose in the present paper is to construct models which combine the interesting features from the above two classes of theories. In fact, we succeed in constructing the first string compactifications with interesting 4d chiral gauge sectors and flux-stabilization of (most) moduli. The models are based on introducing NSNS and RR 3-form fluxes in compactifications of type-IIB theory with D-branes. Chirality on the latter arises from the non-trivial gauge bundles on the D-brane world-volumes (a mechanism related, in the absence of fluxes, to D-brane intersections via T-duality). For concreteness, and also to simplify the discussion of the stability of the configurations, we choose to center on models with subsectors with $\mathcal{N} = 1$ supersymmetry in 4d. In the setup of D-brane with world-volume magnetic fields, a simple suitable background geometry is (an orientifold of) the $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold. However, we expect that the techniques and our new observations are useful in constructing supersymmetric models in other geometries; and also to the construction of non-supersymmetric models with these features. In fact, we provide an explicit construction of a non-supersymmetric compactification with fluxes with a gauge sector of D3-branes at singularities leading to a 3-family SU(5) GUT.

The paper is organized as follows. In section 2 we review the construction of toroidal orientifold compactifications with NSNS and RR 3-form fluxes, and discuss the main points to be addressed in the construction of orbifold models. In section 3 we discuss diverse classes

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of D-brane configurations leading to chiral gauge sectors, and issues arising in the possible introduction of 3-form fluxes for them. We conclude that a simple and flexible setup for model building is that of D-branes with world-volume magnetic fields. In section 4 we review the properties and spectra in configurations of such magnetised D-branes in toroidal orientifold models, and describe the new features when $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projections are included.

In section 5 we construct explicit models (with supersymmetric subsectors) of compacifications with 3-form fluxes and configurations of magnetised D-branes. These models have flux-stabilized moduli and include gauge sectors with realistic features, namely reasonable gauge groups, and several families of charged chiral fermions. These constructions provide the first examples in a presumably rich and intereseting class of realistic models.

In section 6 we describe the main modifications in models with a discrete NSNS B-field on some two-torus. We show that it implies the appearance of positively charged O3- and O7-planes in the configuration, hence reducing the RR tadpole, and potentially modifying the quantization conditions for 3-form fluxes. In section 7 we discuss the T-duality relation of our models with compactifications with D-branes on non-Calabi-Yau geometries, and obtain some of the topological couplings relevant to the construction of the later models. Section 8 contains our final comments.

Appendix A describes an alternative approach to compactifications with chiral gauge sectors and few moduli. We construct explicit (non-supersymmetric) compactifications containing 3-form fluxes and chiral gauge sectors arising from D3-branes at singularities, in an orientifold of the $\mathbf{T}^6/\mathbb{Z}_3$ orbifold. Finally appendix B addresses several subtle issues on the properties of D-branes in the presence of NSNS 3-form fluxes (related to the modifications of the K-theory group classifying D-brane states in the presence of 3-form flux). They involve the need to include additional D-branes in some of the models of the main text (not modifying its chiral spectrum), and the possibility of D-brane instanton mediated transitions between D-branes and fluxes.

After this paper was finished and was being prepared for submission, we noticed [48], which also studies the same class of models.

1.1 The $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold

The explicit models we will discuss are based on (an orientifold of) the $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold with discrete torsion (corresponding to Hodge numbers $(h_{1,1}, h_{2,1}) = (3, 51)$). Let us describe the geometry of the $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold (see [1]). The $\mathbb{Z}_2 \times \mathbb{Z}_2$ generators θ, ω act by

$$\begin{aligned} \theta &: (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \end{aligned} \tag{1.1}$$

on the complex coordinates of $\mathbf{T}^6 = (\mathbf{T}^2)^3$. Namely $z_i \to e^{2\pi i v_i} z_i$ with $v_\theta = (1/2, -1/2, 0)$ and $v_\omega = (0, 1/2, -1/2)$.

The spectrum of type-IIB string theory on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold contains in the untwisted sector the 4d $\mathcal{N} = 2$ supergravity multiplet, the dilaton hypermultiplet, three

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vector multiplets and three hypermultiplets. The moduli in these vector and hypermultiplets correspond to the complex structure and Kahler parameters of the three underlying $\mathbf{T}^2$'s. Components of the metric $G_{i\bar{j}}$ mixing different two-tori are projected out by the orbifold actions. The contribution of untwisted modes to the Hodge numbers of the orbifold is $(h_{1,1}, h_{2,1})_{\rm unt} = (3,3)$.

The orbifold contains twisted sectors of θ, ω and $\theta\omega$. The fixed point set of each twist is given by 16 two-tori, near each of which the Calabi-Yau has a local geometry $\mathbb{C}^2/\mathbb{Z}_2 \times \mathbf{T}^2$. Geometrically, for our choice of discrete torsion, each of these fixed tori leads to two collapsed 3-cycle, given by the $\mathbf{P}_1$ collapsed at the $\mathbb{C}^2/\mathbb{Z}_2$ singularity times the 1-cycles in $\mathbf{T}^2$. The 3-cycles are thus of $(2,1)$ and $(1,2)$ type. The contribution of twisted sectors to the Hodge numbers of the orbifold is $(h_{1,1}, h_{2,1})_{\rm tw.} = (0, 3 \times 16) = (0, 48)$. The Hodge numbers of the orbifold space are therefore $(h_{1,1}, h_{2,1}) = (3, 51)$.

2. Review of fluxes

Compactifications of type-II theories (or orientifolds thereof) with NSNS and RR field strength fluxes have been considered, among others, in [2]–[7]. In this section we review properties of type-IIB compactifications with 3-form fluxes.

2.1 Consistency conditions and moduli stabilization

Type-IIB compactification on a Calabi-Yau threefold X_6 with non-trivial NSNS and RR 3-form field strength backgrounds H_3, F_3 have been extensively studied. In particular the analysis in [3] provided in a very general setup the consistency conditions such fluxes should satisfy. They must obey the Bianchi identities

$$dF_3 = 0 \quad dH_3 = 0 \tag{2.1}$$

and they should be properly quantized, namely for any 3-cycle $\Sigma \subset \mathbf{X}_6$

$$\frac{1}{(2\pi)^2\alpha'} \int_\Sigma F_3 \ \in \mathbb{Z}\,; \qquad \frac{1}{(2\pi)^2\alpha'} \int_\Sigma H_3 \ \in \mathbb{Z} \tag{2.2}$$

The fluxes hence define integer 3-cohomology classes in $H^3(X_6, \mathbb{Z})$.

A subtlety in flux quantization in toroidal orientifolds was noticed in [4, 5], see also section 6. Namely, if flux integrals along some 3-cycle are integer but odd, consistency requires the corresponding 3-cycle to pass through an odd number of exotic O3-planes (studied in [8]). For simplicity we restrict to the case where all flux integrals are even integers.

An important observation in [3] is that, in order to avoid previous no-go theorems about the existence of configurations of fluxes satisfying the equations of motion, it is crucial to include orientifold 3-planes in the compactification, so we consider type-IIB orientifolds with these objects. The simplest way to understand the need of these objects, is to notice the type-IIB supergravity Chern-Simons coupling

$$\int_{M_4 \times X_6} H_3 \wedge F_3 \wedge C_4 \tag{2.3}$$

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where C_4 is the IIB self-dual 4-form gauge potential. This coupling implies that upon compactification the flux background contributes to a tadpole for C_4, with positive coefficient $N_{\rm flux}$[1] (in D3-brane charge units). Moreover, fluxes contribute positively to the energy of the configuration, due to the 2-form kinetic terms. The only way to cancel these tadpoles is to introduce objects with negative RR C_4-charge and negative tension, to cancel both the RR tadpole and also to compensate the vacuum energy of the configuration. Having O3-planes in the configuration, it is natural to consider the possibility of adding N_{Q_3} explicit D3-branes as well. The RR tadpole cancellation constraint hence reads

$$N_{Q_3} + N_{\rm flux} + Q_{O3} = 0 \tag{2.4}$$

we normalize charge such that a D3-brane in covering space has charge +1. With this convention an O3-plane has charge $-1/2$, and

$$N_{\rm flux} = \frac{1}{(4\pi^2\alpha')^2}\int_{X_6} H_3 \wedge F_3 = \frac{1}{(4\pi^2\alpha')^2}\frac{i}{2\phi_I}\int_{X_6} G_3 \wedge \overline{G}_3 \tag{2.5}$$

where ϕ_I is the imaginary part of the IIB complex coupling $\phi = a + i/g_s$, and

$$G_3 = F_3 - \phi H_3 \,. \tag{2.6}$$

Finally, in order to satisfy the equations of motion, the flux combination G_3 must be imaginary self-dual with respect to the Hodge operation defined in terms of the Calabi-Yau metric in X_6

$$*_6\, G_3 = i\, G_3 \,. \tag{2.7}$$

Given these conditions, the analysis in [3] guarantees the existence of a consistent supergravity solution for the different relevant fields in the configuration, metric, and 4-form, which have the form of a warped compactification (similar to a black 3-brane solution, since the same fields are sourced).[2]

We remark that the above condition should not be regarded as an additional constraint on the fluxes. Rather, for a set of fluxes in a fixed topological sector (i.e. in a fixed cohomology class), eq. (2.7) is a condition on the scalar moduli which determine the internal metric. The scalar potential is minimized at points in moduli space where (2.7) is satisfied, while fluxes induce a positive scalar potential at other points. Hence introduction of fluxes leads to a natural mechanism to stabilize moduli. Explicit expressions will be discussed later on, for the moment let us state the result in [3, 4, 5] that generically all complex structure moduli and most Kahler moduli are stabilized by this mechanism.

[1] This is so if we require the flux to preserve the same supersymmetry as the O3-planes. This is implicit in the literature, since fluxes leading to negative RR 4-form charge would lead to uncancelled NSNS tadpoles, and hence to non-Poincare invariant 4d theories.

[2] Interestingly, the above configurations lead, at the classical supergravity level, to 4d Poincare invariant solutions (i.e. vanishing cosmological constant) and flat potential for the overal Kahler parameter, even in the absence of supersymmetry [3], although α' corrections [9] in general spoil this property. Even with supersymmetry, spacetime non-perturbative effects [10] may generate supersymmetry breaking effects and generate a cosmological constant. We will have nothing to say about this familiar problem.

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2.2 Supersymmetry

The conditions for a configurations with 3-form fluxes to preserve some supersymmetry have been studied in [11], and applied in explicit constructions in [4, 5]. Let us review these results.

The 10d $\mathcal{N} = 2$ type-IIB real supersymmetry transformation parameters ϵ_L, ϵ_R, can be gathered into a complex one $\epsilon = \epsilon_L + i\epsilon_R$. It is chiral in 10d, satisfying $\Gamma_{10d}\epsilon = -\epsilon$, with $\Gamma_{10d} = \gamma^0 \dots \gamma^9$. Compactification on $\mathbf{X}_6$ splits this spinor with respect to SO(6) × SO(4) e.g. as

$$\epsilon_L = \xi \otimes u + \xi^* \otimes u^* \tag{2.8}$$

where ξ is a 6d chiral spinor $\Gamma_{6d}\xi = -\xi$, and u is a 4d chiral spinor $\Gamma_{4d}u = u$. For $\mathbf{X}_6$ of generic SU(3) holonomy only one component of ξ is covariantly constant and provides susy transformations in 4d.

On the other hand, the presence of the O3-planes and D3-branes in the background preserves only those ϵ satisfying

$$\epsilon_R = -\gamma^4 \dots \gamma^9 \epsilon_L \,. \tag{2.9}$$

Such spinors are of the form $\epsilon = 2\xi \otimes u$.

The conditions for a flux to preserve a supersymmetry associated to a particular spinor component of ξ are [11]

$$G\xi = 0\,; \qquad G\xi^* = 0\,; \qquad G\gamma^m\xi^* = 0 \tag{2.10}$$

where $G = \frac{1}{6}G_{mnl}\gamma^{[m}\gamma^n\gamma^{l]}$.

To understand this a bit better, let us introduce complex coordinates $z^i, \overline{z}^i$, where the gamma matrix algebra reads

$$\left\{\gamma^i, \gamma^j\right\} = \left\{\gamma^{\overline{i}}, \gamma^{\overline{j}}\right\} = 0\,; \qquad \left\{\gamma^i, \gamma^{\overline{j}}\right\} = \delta^{i\overline{j}} \tag{2.11}$$

Introducing the highest weight state ξ_0 satisfying $\gamma^{\overline{i}}\xi_0 = 0$, the spinor representation is

State	SO(6) weight	State	SO(6) weight
ξ_0	$\frac{1}{2}(+++)$	$\gamma^i\xi_0$	$\frac{1}{2}(\underline{-++})$
$\gamma^1\gamma^2\gamma^3\xi_0$	$\frac{1}{2}(---)$	$\gamma^i\gamma^j\xi_0$	$\frac{1}{2}(\underline{--+})$

The O3-planes preserve ξ_0 and $\gamma^{ij}\xi_0$. Of these, a general Calabi-Yau (on which z_i are complex coordinates) preserves only ξ_0, since it is SU(3) invariant.

The conditions that a given flux preserves ξ_0, can be described geometrically [11, 4, 5] as

a) G_3 is of type $(2,1)$ in the corresponding complex structure

b) $G_3 \wedge J = 0$ where J is the Kahler form.

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For explicit discussion of these conditions see below. Notice that a G_3 flux which is not $(2,1)$ in a complex structure, may still be supersymmetric if it preserves other spinor ξ_0' (although it does not preserve ξ_0). In such case, G_3 would be of type $(2,1)$ in a different complex structure where ξ_0' is the spinor annihilated by the new $\gamma^{\overline{i}'}$.

Since the techniques to find consistent (possibly supersymmetric) fluxes at particular values of the stabilized moduli (and vice versa) have been discussed in the literature, we will not dwelve into their discussion.

In a generic Calabi-Yau compactification, where the holonomy is SU(3) but not a subgroup of SU(2)), there is a unique component of the spinor which is covariantly constant with respect to the spin connection. In this kind of situations, there is a preferred complex structure, that in which that spinor is annihilated by $\gamma^{\overline{i}}$. We will be interested in fluxes that preserve that spinor (so we call it ξ_0) and then G_3 should be $(2,1)$ in that complex structure.

In toroidal compactifications, the holonomy preserves several spinors, so we can play with different complex structures. In toroidal orbifolds, the orbifold projections in general project out some spinor components, leaving others invariant (just one if the orbifold group is in SU(3) but not in SU(2)). In such case some complex structure is preferred, in analogy with the Calabi-Yau case.

2.3 Fluxes in the $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

We will be interested in discussing compactification on orbifolds with fluxes. The orbifold projection introduces some modifications with respect to the above toroidal case.

i) Only some subset of fluxes is invariant under the orbifold projection.

ii) The orbifold space may contain closed cycles which are not closed in the covering space. For instance, taking a $\mathbf{T}^4$ parametrized by x_i with identifications $x_i \simeq x_i + 1$, modded out by $x_i \to -x_i$, one such 2-cycle is provided by points (x_1, x_2) with $0 \leq x_1 < 1$, $0 \leq x_2 < 1/2$. In general, the volume of such cycles is given by the volume of some cycle in the covering space, divided by the order of the orbifold. Hence if they exist they impose more restrictive quantization constraints of the fluxes.[3]

iii) In addition, orbifolds contain new cycles, collapsed at the singular points, which are not present in the parent torus. Thus one has to ensure proper quantization of the field strength fluxes not just over the cycles in the parent torus, but also over the cycles collapsed at the singularities.

iv) Orbifolds contain additional moduli associated to the twisted sectors. In general, these scalar moduli also appear in the complete flux-induced scalar potential. In working at the orbifold limit, one should make sure that this point in geometric moduli space indeed corresponds to a minimum of the potential, with respect to these scalars.

[3] This should be more properly understood as imposing quantization of the flux over a basis of cycles in the quotient space, whose elements in general involve linear combinations of untwisted and collapsed cycles. We thank R. Blumenhagen for discussion on this point.

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v) As mentioned above, supersymmetry of the configuration requires the fluxes to preserve the spinor component which is invariant under the orbifold action.

The systematic analysis of these features for arbitrary orbifolds seems rather difficult, so we prefer to center on a particular example. We will be interested in the particular case of type-IIB on $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ modded out by ΩR. The geometry of the orbifold has been described in section 1.1.

Let us discuss the above issue in detail in the present setup.

iii) We consider fluxes only for untwisted NSNS and RR fields (constant fluxes). This allows to apply the techniques developed for toroidal orientifolds, and moreover ensures that their integral over 3-cyles collapsed at the singular points is zero. The integral would be proportional to the correlator $\langle \mathcal{O}_T \mathcal{O}_G \rangle$ of the twisted form and 2-form vertex operators, which vanishes due to mismatch of discrete charge under the $\mathbb{Z}_2$ quantum symmetry. This implies that these fluxes are properly quantized with respect to collapsed cycles.

i) NSNS and RR 3-form fluxes invariant under the orbifold actions should involve one leg on each two torus.

iv) We will ensure the fluxes are supersymmetric at the point in CY moduli space corresponding to the orbifold limit, so it is guaranteed that the orbifold limit is a minimum of the scalar potential. In addition for the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold (with our choice of discrete torsion) twisted sector moduli correspond to complex structure deformations. Following [3], we expect all these moduli to be stabilized (at the orbifold point in moduli space) by the fluxes. Hence the orbifolds we discuss do not contain additional moduli from twisted sectors.

v) There is a preferred spinor, surviving the orbifold projection (CY holonomy), given by $\xi_0 = 1/2(+++)$. Hence to build susy fluxes we center on fluxes preserving the corresponding supersymmetry. Namely, of $(2,1)$ kind in the complex structure given by (z_1, z_2, z_3).

ii) Unfortunately, the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold of $\mathbf{T}^6/\Omega R$ contains a closed cycle, $0 \leq x_i \leq 1/2$ for $i = 4, 6, 8$, which is not closed in $\mathbf{T}^6/\Omega R$. This cycle requires the fluxes over toroidal cycles to be quantized to multiples of 8. The latter oversaturate the RR tadpole from the O3-planes and require the introduction of antibranes in the configuration, which is therefore non-supersymmetric. For simplicity we will preserve $\mathcal{N} = 1$ supersymmetry in the chiral sector of the theory, and simply add anti-D3-branes to cancel the excess of RR charge arising from the flux.

An additional advantage of our orbifold models is that the orbifold projection eliminates some of the (non-stabilized) Kahler moduli present in toroidal orientifolds. Hence, in a sense, our models stabilize (almost) all moduli by a combination of orbifold projections and NSNS and RR fluxes.

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3. Philosophy of our approach

The models considered in [5, 4] succeed in leading to $\mathcal{N} = 1$ or non-supersymmetric low-energy theories, with stabilization of most moduli. However, they are unrealistic in that they are automatically non-chiral, since the only gauge sectors live on parallel D3-branes whose low-energy spectra are non-chiral.[4]

We are interested in constructing models containing gauge sectors with charged chiral fermions, and with a bulk with flux-induced moduli stabilization. There are several possibilities to do this, corresponding to the different ways to build configurations of D-branes containing chiral fermions. In this paper we center on supersymmetric model building, leaving several interesting non-supersymmetric setups for future work.

D3-branes at singularities. One possibility is to use compactification varieties containing singular points, e.g. orbifold singularities. Locating D3-branes at the singularity leads to chiral gauge sectors, with low energy spectrum given by a quiver diagram [12].[5] A simple possibility would be to consider type-IIB theory on $\mathbf{T}^6/\mathbb{Z}_3$ with the ΩR orientifold projection introduced above. This is particularly promising, since it is the simplest orbifold which can lead to three families in the sector of D3-branes at singularities. However, it is not possible to obtain $\mathcal{N} = 1$ supersymmetric models in this setup, for the following reason. In the complex structure where the spinor invariant under $\mathbb{Z}_3$ satisfies $\gamma^{\bar{i}}\xi_0 = 0$, the $\mathbb{Z}_3$ orbifold action reads

$$\theta : (z_1, z_2, z_3) \to \left(e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{-4\pi i/3} z_3\right). \tag{3.1}$$

Fluxes preserving the same spinor ξ_0 should be of type $(2,1)$ in this complex structure, namely linear combinations of

$$d\bar{z}^1 dz^2 dz^3\,; \qquad dz^1 d\bar{z}^2 dz^3\,; \qquad dz^1 dz^2 d\bar{z}^3\,. \tag{3.2}$$

Such fluxes are not invariant under the orbifold action, and cannot be turned on. In other words, the only possible fluxes are not supersymmetric. The same problem arises for other promising orientifolds, like $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. So we will not pursue the construction of $\mathcal{N} = 1$ susy models in the setup of D3-branes at singularities.

An alternative would be to give up supersymmetry, and build $\mathcal{N} = 0$ models of this kind. An amusing possibility is that of models where the D-brane sector preserves some supersymmetry, while the closed string sector is non-supersymmetric due to the combination of fluxes and orbifold action. We provide an example of this kind in appendix A.

Intersecting D6-branes. Much progress has been made in D-brane model building using type-IIA D6-branes wrapped on intersecting 3-cycles in an internal space (e.g. [14]–[20]).[6] However it is difficult to introduce NSNS and RR field strength fluxes in those

[4] The D3-brane world-volume spectra are at best $\mathcal{N} = 1, 0$ deformations of $\mathcal{N} = 4$ theories, by flux-induced operators breaking partially or totally the world-volume supersymmetry

[5] For model building in this setup with no field strength fluxes, see [13].

[6] See [21] for early work leading to non-chiral models, and [22] for reviews

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setups; the difficulties can be seen from different perspectives. The models usually contain O6-planes; this implies that to avoid the usual no-go theorems about turning on fields strength fluxes in supergravity we need combinations of fluxes which source the RR 7-form. A possible combination is the RR 0-form field strength of type-IIA (i.e. a cosmological constant of massive type-IIA) and the NSNS 3-form field strength. This combination of fluxes has not been studied in the literature, so it is not a convenient starting point.

One may think that T-dualizing three times a model with O3-planes and RR and NSNS fluxes would yield the desired configurations with O6-planes. However, T-duality acts in a very non-trivial way on H_{NSNS}, transforming it into non-trivial components of the T-dual metric, which is no longer Calabi-Yau [23, 24, 25]. The final configuration indeed would contain fluxes (RR and 'metric fluxes') which source the RR 7-form, and would lead to Poincare invariant 4d models (consistently with T-duality). However, an analysis of the possible 3-cycles on which to wrap D6-branes in the resulting complicated geometries is lacking.

Magnetised D-branes. We propose to take a different route. Although it is not often explicitly stated, it is also possible to obtain chiral fermions from wrapped D-branes, if the geometry of the wrapped manifold or the topology of the internal world-volume gauge bundle are non-trivial. The chiral fermions arise from the Kaluza-Klein reduction of the higher-dimensional worldvolume fermions if the index of the corresponding Dirac operator is non-zero.

We will center on a particular realization of this, corresponding to type IIB compactified on $\mathbf{T}^6$ (with an additional ΩR orientifold, and possibly orbifold projections), with configurations of D9-branes spanning all of spacetime, namely wrapped on $\mathbf{T}^6$ and with non-trivial gauge bundles on $\mathbf{T}^6$. The bundles are given by constant magnetic fields on each of the $\mathbf{T}^2$, so we call them magnetised D-brane configurations. They have been considered in [27] in the absence of closed string field strength fluxes.

In the absence of fluxes, these configurations are related by T-duality to configurations of intersecting D6-branes, so that any model of the latter kind can be translated very easily (see [14, 28]) to a magnetised D9-brane setup. In particular, this shows that there exist $\mathcal{N} = 1$ supersymmetric models of magnetised D9-branes containing chiral fermions, by simply T-dualizing the models of intersecting D6-branes in [19].

The advantage of using the magnetised D9-brane picture with O3-planes is that it is now straightforward to include NSNS and RR fluxes in the configuration, by applying the tools reviewed above (for the situation without D-branes). Notice that in the T-dual version of intersecting branes this corresponds to turning on a complicated set of NSNS, RR and metric fluxes. For the class of fluxes we consider, the picture of magnetised D9-branes is more useful[7]

In the following section we review magnetised D9-brane configurations without NSNS and RR fluxes. The new ingredients due to turning on the latter are described in section 5.

[7] For the most general set of fluxes, the generalized mirror symmetry in [23] dictates the translation between configurations of A-type branes (D6-branes on 3-cycles) and B-type branes (D9-branes with bundles). In this general situation both pictures are in principle equally complicated.

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4. Magnetised D-branes

In this section we review configurations of magnetised D9-branes before the introduction of fluxes. We first consider the case of toroidal compactifications, and subsequently incorporate orientifold projections and orbifold projections. The models, in the absence of bulk fluxes, are T-dual to the models of intersecting D6-branes in toroidal compactifications [15], toroidal orientifolds [14] and orbifolds [19].

4.1 Magnetised D-branes in toroidal compactifications

We start with the simple case of toroidal compactification, with no orientifold projection. Consider the compactification of type-IIB theory on $\mathbf{T}^6$, assumed factorizable.[8]

We consider sets of N_a D9-branes, labelled D9$_a$-branes, wrapped m_a^i times on the i^{th} 2-torus $(\mathbf{T}^2)_i$ in $\mathbf{T}^6$, and with n_a^i units of magnetic flux on $(\mathbf{T}^2)_i$. Namely, we turn on a world-volume magnetic field F_a for the center of mass U$(1)_a$ gauge factor, such that

$$m_a^i \frac{1}{2\pi} \int_{\mathbf{T}_i^2} F_a^i = n_a^i \,. \tag{4.1}$$

Hence the topological information about the D-branes is encoded in the numbers N_a and the pairs (m_a^i, n_a^i).[9]

We can include other kinds of lower dimensional D-branes using this description. For instance, a D7-brane (denoted D7$_{(i)}$) sitting and a point in $\mathbf{T}_i^2$ and wrapped on the two remaining two-tori (with generic wrapping and magnetic flux quanta) is described by $(m^i, n^i) = (0, 1)$ (and arbitrary (m^j, n^j) for $j \neq i$); similarly, a D5-brane (denoted D5$_{(i)}$) wrapped on $\mathbf{T}_i^2$ (with generic wrapping and magnetic flux quanta) and at a point in the remaining two 2-tori is described by $(m^j, n^j) = (0, 1)$ for $j \neq i$; finally, a D3-brane sitting at a point in $\mathbf{T}^6$ is described by $(m^i, n^i) = (0, 1)$ for $i = 1, 2, 3$. This is easily derived by noticing that the boundary conditions for an open string ending on a D-brane wrapped on a two-torus with magnetic flux become Dirichlet for (formally) infinite magnetic field.

D9-branes with world-volume magnetic fluxes are sources for the RR even-degree forms, due to their worldvolume couplings

$$\int_{D9_a} C_{10} \,; \qquad \int_{D9_a} C_8 \wedge \mathrm{tr}\, F_a \,; \qquad \int_{D9_a} C_6 \wedge \mathrm{tr}\, F_a^2 \,; \qquad \int_{D9_a} C_4 \wedge \mathrm{tr}\, F_a^3 \,. \tag{4.2}$$

Consistency of the configuration requires RR tadpoles to cancel. Following the discussion in [15], leads to the conditions

$$\sum_a N_a m_a^1 m_a^2 m_a^3 = 0$$

[8] Without orbifold projections, this requires a constrained choice of fluxes, stabilizing moduli at values corresponding to a factorized geometry. In our orbifolds below, such moduli are projected out by the orbifold, and hence are simply absent.

[9] Notice the change of roles of n and m as compared with other references. This however facilitates the translation of models in the literature to our language.

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$$\sum_a N_a m_a^1 m_a^2 n_a^3 = 0 \qquad \text{and permutations of } 1,2,3$$
$$\sum_a N_a m_a^1 n_a^2 n_a^3 = 0 \qquad \text{and permutations of } 1,2,3$$
$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 0\,. \tag{4.3}$$

Which amounts to cancelling the D9-brane charge as well as the induced D7-, D5- and D3-brane charges.

Introducing for the i^{th} 2-torus the even homology classes $[\mathbf{0}]_i$ and $[\mathbf{T}^2]_i$ of the point and the two-torus, the vector of RR charges of the one D9-brane in the a^{th} stack is

$$[\mathbf{Q}_a] = \prod_{i=1}^{3} \left(m_a^i \left[\mathbf{T}^2\right]_i + n_a^i [\mathbf{0}]_i \right). \tag{4.4}$$

The RR tadpole cancellation conditions read

$$\sum_a N_a [\mathbf{Q}_a] = 0\,. \tag{4.5}$$

The conditions that two sets of D9-branes with worldvolume magnetic fields F_a^i, F_b^i preserve some common supersymmetry can be derived from [29]. Indeed, it is possible to compute the spectrum of open strings stretched between them and verify that it is supersymmetric if

$$\Delta_{ab}^1 \pm \Delta_{ab}^2 \pm \Delta_{ab}^3 = 0 \tag{4.6}$$

for some choice of signs. Here

$$\Delta_i = \arctan\left[(F_a^i)^{-1}\right] - \arctan\left[(F_b^i)^{-1}\right] \tag{4.7}$$

and

$$F_a^i = \frac{n_a^i}{m_a^i R_{x_i} R_{y_i}} \tag{4.8}$$

which follows from (4.1).

The spectrum of massless states is easy to obtain. The sector of open strings in the aa sector leads to $\mathrm{U}(N_a)$ gauge bosons and superpartners with respect to the 16 supersymmetries unbroken by the D-branes. In the $ab + ba$ sector, the spectrum is given by I_{ab} chiral fermions in the representation $(N_a, \overline{N}_b)$, where

$$I_{ab} = [\mathbf{Q}_a] \cdot [\mathbf{Q}_b] = \prod_{i=1}^{3} \left(n_a^i m_b^i - m_a^i n_b^i \right) \tag{4.9}$$

is the intersection product of the charge classes, which on the basic classes $[\mathbf{0}]_i$ and $[\mathbf{T}^2]_i$ is given by the bilinear form

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{4.10}$$

The above multiplicity can be computed using the α'-exact boundary states for these D-branes [14], or from T-duality with configurations of intersecting D6-branes. We now

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provide an alternative derivation which remains valid in more complicated situations where the worldsheet theory is not exactly solvable. Consider for simplicity a single two-torus. We consider two stacks of N_a and N_b branes wrapped m_a and m_b times, and with n_a, n_b monopole quanta. Consider the regime where the two-torus is large, so that the magnetic fields are diluted and can be considered a small perturbation around the vacuum configuration. In the vacuum configuration, open strings within each stack lead to a gauge group $\mathrm{U}(N_a m_a)$ and $\mathrm{U}(N_b m_b)$ respectively, which is subsequently broken down to $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$ by the monopole background, via the branching

$$\mathrm{U}(N_a m_a) \times \mathrm{U}(N_b m_b) \to \mathrm{U}(N_a)^{m_a} \times \mathrm{U}(N_b)^{m_b} \to \mathrm{U}(N_a) \times \mathrm{U}(N_b)\,. \tag{4.11}$$

Open ab strings lead to a chiral 10d fermion transforming in the bifundamental $(\Box_a, \overline{\Box}_b)$ of the original $\mathrm{U}(N_a m_a) \times \mathrm{U}(N_b m_b)$ group. Under the decomposition (4.11) the representation splits as

$$(\Box_a, \overline{\Box}_b) \to (\underline{\Box_a, \ldots}; \underline{\overline{\Box}_b, \ldots}) \to m_a m_b (\Box_a, \overline{\Box}_b) \tag{4.12}$$

The 8d theory contains chiral fermions arising from these, because of the existence of a nonzero index for the internal Dirac operator (coupled to the magnetic field background). The index is given by the first Chern class of the gauge bundle to which the corresponding fermions couples. Since it has charges $(+1, -1)$ under the a^{th} and b^{th} U(1)'s, the index is

$$\mathrm{ind}\not{D}_{ab} = \int_{\mathbf{T}^2} (F_a - F_b) = \frac{n_a}{m_a} - \frac{n_b}{m_b}\,. \tag{4.13}$$

Because of the branching (4.12), a single zero mode of the Dirac operator gives rise to $m_a m_b$ 8d chiral fermions in the $(\Box_a, \overline{\Box}_b)$ of $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$. The number of chiral fermions in the 8d theory in the representation $(\Box_a, \overline{\Box}_b)$ of the final group is given by $m_a m_b$ times the index, namely

$$I_{ab} = m_a m_b \int_{\mathbf{T}^2} (F_a - F_b) = n_a m_b - m_a n_b\,. \tag{4.14}$$

The result (4.9) is a simple generalization for the case of compactification on three two-tori.

An important property about these chiral fields is that they are localized at points in the internal space. From the string theory viewpoint this follows because boundary conditions for open strings with endpoints on D-branes with different magnetic fields require the absense of center of mass zero mode in the worldsheet mode expansion. From the low energy effective theory viewpoint, this follows because such strings behave as charged particles in a magnetic field. From elementary quantum mechanics, such particles feel a harmonic oscillator potential and are localized in the internal space. Excited states in the harmonic oscillator system (Landau levels) correspond to stringy oscillator (gonions [15] in T-dual picture).

Notice that the field theory argument to obtain the spectrum is valid only in the large volume limit. However, the chirality of the resulting multiplets protects the result, which can therefore be extended to arbitrarily small volumes. This kind of argument will be quite useful in the more involved situation with closed string field strength fluxes, where we do not have a stringy derivation of the results.

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4.2 Magnetised D-branes in toroidal orientifolds

We are interested in adding orientifold planes into this picture, since they are required to obtain supersymmetric fluxes. Consider type-IIB on $\mathbf{T}^6$ (with zero NSNS B-field) modded out by ΩR, with $R : x_m \to -x_m$. This introduces 64 O3-planes, which we take to be all $O3^-$ (see section 6 for subtleties). It also requires the D9-brane configuration to be $\mathbb{Z}_2$ invariant. Namely, for the N_a D9$_a$-brane with topological numbers (m_a^i, n_a^i) we need to introduce their N_a ΩR images D9$_{a'}$ with numbers $(-m_a^i, n_a^i)$.

The RR tadpole cancellation conditions read

$$\sum_a N_a [\mathbf{Q}_a] + \sum_a N_a [\mathbf{Q}_{a'}] - 32\,[\mathbf{Q}_{O3}] = 0 \tag{4.15}$$

with $[\mathbf{Q}_{O3}] = [\mathbf{0}]_1 \times [\mathbf{0}]_2 \times [\mathbf{0}]_3$. More explicitly

$$\begin{aligned} \sum_a N_a m_a^1 m_a^2 n_a^3 &= 0 \qquad \text{and permutations of } 1,2,3 \\ \sum_a N_a n_a^1 n_a^2 n_a^3 &= 16\,. \end{aligned} \tag{4.16}$$

Namely, cancellation of induced D7- and D3-brane charge. Notice that there is no net D9- or D5-brane charge, in agreement with the fact that the orientifold projection eliminates the corresponding RR fields.[10]

The rules to obtain the spectrum are similar to the above ones, with the additional requirement of imposing the ΩR projections. This requires a precise knowledge of the ΩR action of the different zero mode sectors (in field theory language, on the harmonic oscillator groundstates for chiral fermions). The analysis is simplest in terms of the T-dual description, where it amounts to the geometric action of the orientifold on the intersection points of the D-branes. The result, which is in any case derivable in our magnetised brane picture, can be taken from [14].

The aa sector is mapped to the $a'a'$ sector, hence suffers no projection[11] We obtain a 4d U(N_a) gauge group, and superpartners with respect to the $\mathcal{N} = 4$ supersymmetry unbroken by the brane.

[10] There is also an additional discrete constraint, previously unnoticed in the literature, which we would like to point out. It follows from a careful analysis of K-theory D-brane charge in the presence of orientifold planes. Following [30], the charge of D5-branes wrapped on some $\mathbf{T}^2$ in the presence of O3-planes is classified by a real K-theory group which is $\mathbb{Z}_2$. This statement is T-dual to the fact that D7-brane charge is $\mathbb{Z}_2$ valued in type-I theory. Following [31] RR tadpole cancellation requires cancellation of the K-theory D-brane charge. Hence the total induced D5-brane charge on the D9$_a$-branes (without images) must be even in the above configurations. This amounts to the condition

$$\sum_a N_a m_a^1 n_a^2 n_a^3 = \text{even and permutations of } 1,2,3\,. \tag{4.17}$$

The condition is non-trivial, and models satisfying RR tadpole conditions in homology, but violating RR tadpole conditions in K-theory can be constructed [32]. Such models are inconsistent, as can be made manifest by introducing a D7-brane probe, on which world-volume the inconsistency manifests as a global gauge anomaly [31]. The condition is however happily satisfied by models in the literature, and also in our examples below.

[11] We do not consider branes for which $a = a'$ here; they will be taken care of explicitly in the examples below.

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The $ab+ba$ sector is mapped to the $b'a'+a'b'$ sector, hence does not suffer a projection. We obtain I_{ab} 4d chiral fermions in the representation $(\square_a, \overline{\square}_b)$. Plus additional scalars which are massless in the susy case, and tachyonic or massive otherwise.

The $ab'+b'a$ sector is mapped to the $ba'+a'b$. It leads to $I_{ab'}$ 4d chiral fermions in the representation $(\square_a, \square_b)$ (plus additional scalars).

The $aa'+a'a$ sector is invariant under ΩR, so suffers a projection. The result is $n_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ and $n_{\square\square}$ 4d chiral fermions in the $\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$, $\square\square_a$ representations, resp, with

$$\begin{aligned} n_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} &= \frac{1}{2}(I_{aa'} + 8I_{a,O3}) = -4\, m_a^1 m_a^2 m_a^3 \left(n_a^1 n_a^2 n_a^3 + 1\right) \\ n_{\square\square} &= \frac{1}{2}(I_{aa'} - 8I_{a,O3}) = -4\, m_a^1 m_a^2 m_a^3 \left(n_a^1 n_a^2 n_a^3 - 1\right) \end{aligned} \tag{4.18}$$

where $I_{a,O3} = [\mathbf{Q}_a]\cdot[\mathbf{Q}_{O3}]$.

4.3 Magnetised D-branes in the $\mathbf{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ orbifold

Finally, we will be interested in models with orbifold and orientifold actions. In particular, consider type-IIB on the orbifold $\mathbf{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ described in section 1.1, modded out by ΩR. The model contains (for zero B-field, see section 6) 64 O3-planes (with $-1/2$ units of D3-brane charge), and 4 O7$_i$-planes (with -8 units of D7$_i$-brane charge), transverse to the i^{th} two-torus. Their total charges are given by -32 times the classes

$$\begin{aligned} [\mathbf{Q}_{O3}] &= [\mathbf{0}_1]\times[\mathbf{0}_2]\times[\mathbf{0}_3]\,; \qquad [\mathbf{Q}_{O7_1}] = -[\mathbf{0}_1]\times[(\mathbf{T}^2)_2]\times[(\mathbf{T}^2)_3] \\ [\mathbf{Q}_{O7_2}] &= -[(\mathbf{T}^2)_1]\times[\mathbf{0}_2]\times[(\mathbf{T}^2)_3]\,; \qquad [\mathbf{Q}_{O7_3}] = -[(\mathbf{T}^2)_1]\times[(\mathbf{T}^2)_2]\times[\mathbf{0}_3] \end{aligned} \tag{4.19}$$

where the signs are related to the specific signs in the definition of the $\mathbb{Z}_2\times\mathbb{Z}_2$ action. We define $[\mathbf{Q}_{Op}] = [\mathbf{Q}_{O3}] + [\mathbf{Q}_{O7_1}]+[\mathbf{Q}_{O7_2}]+[\mathbf{Q}_{O7_3}]$. The RR charge is cancelled using magnetised D9-branes and their orientifold images (the orbifold projection maps each stack of D9-branes to itself), which carry just induced D7$_i$- and D3-brane charges. The RR tadpole conditions read

$$\sum_a N_a[\mathbf{Q}_a] + \sum_a N_a[\mathbf{Q}_{a'}] - 32\,[\mathbf{Q}_{Op}] = 0 \tag{4.20}$$

The models with magnetised D9-branes in this orientifold are T-dual to those in [19], whose main features are easily translated. The spectrum can be computed using the above techniques, taking care of the additional orbifold projections on the spectrum, or equivalently translated from [19]. The result is shown in table 1, where $I_{a,Op} = [Q_a]\cdot[Q_{Op}]$.

Sector	Representation
aa	U($N_a/2$) vector multiplet 3 Adj. chiral multiplets
$ab+ba$	I_{ab} $(\square_a, \overline{\square}_b)$ fermions
$ab'+b'a$	$I_{ab'}$ $(\square_a, \square_b)$ fermions
$aa'+a'a$	$\frac{1}{2}(I_{aa'} - 4I_{a,Op})$ $\square\square$ fermions $\frac{1}{2}(I_{aa'} + 4I_{a,Op})$ $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ fermions

Table 1: General chiral spectrum on generic magnetised D9$_a$-branes in the ΩR orientifold of $\mathbf{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$. The models may contain additional non-chiral pieces which we ingnore here. In supersymmetric situations, scalars combine with the fermions given above to form chiral supermultiplets.

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5. The models

5.1 Construction of the models

The models we are going to construct have the following structure. We consider type-IIB theory on $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, where the orbifold twists act as

$$\begin{aligned} \theta &: (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \end{aligned} \tag{5.1}$$

and mod out by ΩR, where

$$R : (z_1, z_2, z_3) \to (-z_1, -z_2, -z_3)\,. \tag{5.2}$$

As explained above, the model contains[12] 64 O3-planes and 4 O7$_i$-planes (localized at points in the i^{th} two-torus, and spanning the remaining two). The orbifold and orientifold projections preserve the spinor satisfying $\gamma^{\bar{i}}\xi = 0$ in the above complex coordinates.

We include a combination of NSNS and RR 3-form field strength fluxes invariant under the orbifold/orientifold actions, and which preserve the same spinor ξ_0. This means that the corresponding 3-form G_3 is of type $(2,1)$ in the above complex coordinates, and involves all three complex coordinates. The general form of such flux is[13]

$$G_3 = g_1\, d\overline{z}_1 dz_2 dz_3 + g_2\, dz_1 d\overline{z}_2 dz_3 + g_3\, dz_1 dz_2 d\overline{z}_3 \tag{5.3}$$

To verify that the flux is supersymmetric, it must also satisfy the primitivity condition $G_3 \wedge J = 0$. In fact, the general Kahler form of $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ is factorizable

$$J = J_1 dz_1 d\overline{z}_1 + J_2 dz_2 d\overline{z}_2 + J_3 dz_3 d\overline{z}_3 \tag{5.4}$$

since the off-diagonal pieces of the metric are projected out by the orbifold action. Hence the above flux is automatically primitive and thus supersymmetric. This also implies that the Kahler parameters of the three two-tori are not stabilized and have flat potential.

As discussed quite generally in [3, 4, 5], the flux stabilizes most geometric moduli. This follows because fluxes are quantized, hence the local values of the field strengths depend on moduli (controlling the sizes of cycles). Hence only for particular values of the moduli the minimization of energy conditions are obeyed by the local values of the field strengths. In our expression above, the stabilization of moduli arises in a formally different way: we have already ensured that the local field strength minimize the energy (and in fact determine a supersymmetric vacuum), but only for very particular values of the moduli the fluxes turn out to be quantized. These are the values for which the moduli are stabilized by the fluxes.

The conditions of proper quantization are easy to obtain (but in general difficult to solve systematically). The periods of dz_i, $d\overline{z}_i$ over the basis 1-cycles $[a_i]$, $[b_i]$, in the two-tori are

$$\int_{[a_i]} dz_j = \delta_{ij}\,; \qquad \int_{[b_i]} dz_j = \tau_i \delta_{ij}\ (\text{no sum})\,. \tag{5.5}$$

[12] Assuming no discrete B-field, see section 6.

[13] In what follows we absorb the normalization factor $1/(4\pi^2\alpha')$ in the definition of the fluxes.

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We also have

$$F_3 = -\frac{\overline{\phi} G_3 - \phi \overline{G}_3}{\phi - \overline{\phi}}\,; \qquad H_3 = -\frac{G_3 - \overline{G}_3}{\phi - \overline{\phi}}\,. \tag{5.6}$$

Hence, the proper quantization of F_3, H_3 on the different 3-cycles requires

$$\begin{aligned} &\int_{[a_1]\times[a_2]\times[a_3]} F_3 \in 8\times\mathbb{Z}\,; \qquad \int_{[a_1]\times[a_2]\times[b_3]} F_3 \in 8\times\mathbb{Z} \quad \text{and perms.} \\ &\int_{[b_1]\times[b_2]\times[b_3]} F_3 \in 8\times\mathbb{Z}\,; \qquad \int_{[b_1]\times[b_2]\times[a_3]} F_3 \in 8\times\mathbb{Z} \quad \text{and perms.} \end{aligned} \tag{5.7}$$

and similarly for H_3. This determines the values at which untwisted complex structure moduli stabilize. The equations are difficult to solve in general, and different tecniques to construct consistent supersymmetric fluxes have been described in the literature [5]. In what follows we will simply choose some particular solutions interesting for our applications.

On the other hand, the fluxes were known from the start to be properly quantized on twisted 3-cycles, when twisted moduli sit at the orbifold point in moduli space. This guarantees that the point in moduli space corresponding to the orbifold geometry is a minimum of the twisted scalar potential. From general analysis [3], we moreover expect that these additional moduli are stabilized at values corresponding to the orbifold geometry, although it would be more involved to verify this more explicitly (in particular it would require an expression of properly quantized fluxes as a funcion of the complex structure deformation of the singularities).

Therefore, the only moduli remaining in the configuration are the Kahler parameters for each of the two-tori. In our models below we will make special choices which make the chiral sector of D9-brane configurations supersymmetric (although this does not imply that these parameters are stabilized [19]).

The contribution from such flux to the C_4 RR tadpole is

$$N_{\text{flux}} = \frac{4\tau_{1,I}\tau_{2,I}\tau_{3,I}}{\phi_I}\left(|g_1|^2 + |g_2|^2 + |g_3|^2\right) \tag{5.8}$$

The remaining RR tapdole for C_4 as well as for the RR 8-forms, must be cancelled by the addition of D-branes. We introduce a set of N_a D9$_a$-branes, with wrapping numbers and world-volume magnetic fluxes given by (m_a^i, n_a^i). Their RR charges are encoded in an even homology class $[\mathbf{Q}]$ of the form (4.4). For each O-plane and the flux, we have the charges (4.19) and

$$[\mathbf{Q}_{\text{flux}}] = N_{\text{flux}}[\mathbf{0}_1]\times[\mathbf{0}_2]\times[\mathbf{0}_3]\,. \tag{5.9}$$

The RR tadpole cancellation conditions for them read

$$[\mathbf{Q}_{\text{tot.}}] = \sum_a N_a[\mathbf{Q}_a] + \sum_{a'} N_a[\mathbf{Q}_{a'}] + [\mathbf{Q}_{\text{flux}}] - 32\,[\mathbf{Q}_{Op}] = 0 \tag{5.10}$$

or more explicitly

$$\sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2}N_{\text{flux}} - 16 = 0$$

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$$\sum_a N_a n_a^1 m_a^2 m_a^3 + 16 = 0$$
$$\sum_a N_a m_a^1 n_a^2 m_a^3 + 16 = 0$$
$$\sum_a N_a m_a^1 m_a^2 n_a^3 + 16 = 0\,. \tag{5.11}$$

Once these conditions are satisfied, a consistent model results.

We will also require that the chiral part of the D-brane configuration preserves the $\mathcal{N} = 1$ supersymmetry unbroken by the orbifold/orientifold, and by the fluxes. The condition on the worldvolume magnetic fields is

$$\sum_i \arctan \frac{m_a^i A_i}{n_a^i} = 0\,. \tag{5.12}$$

When taken for a fixed set of integers (m_a^i, n_a^i), it turns into a constraint on the Kahler moduli of the orbifold. For a small number of D-brane stacks, they can be satisfied by simply adjusting the Kahler parameters of the three two-tori, which are not fixed by the flux-induced potential, as discussed above. For larger numbers, the conditions typically overconstrain the Kahler parameters and the corresponding D-brane configuration cannot be made supersymmteric.[14]

Two subtle points arise in the above construction, related to some potential peculiar features of D-branes (concretely, the K-theory group classifying D-brane charges) in the presence of NSNS 3-form fields. They are discussed in the appendix B, with the outcome that their presence can be ignored in the following discussion.

5.2 The spectrum

Let us describe the corresponding spectrum. Clearly, the presence of the NSNS and RR field strength fluxes does not allow for an α'-exact quantization of the 2d worldsheet theory. Therefore the spectrum can only be described in the supergravity approximation, namely for large internal volumes. In this regime, the fluxes are dilute and their effect is small and under control. For some quantities, protected by supersymmetry and/or chirality, we are allowed to extrapolate the large volume result to the small volume regimes[15]

The discussion of the closed string sector is as in models without D-branes [3, 4, 5]. We obtain the 4d $\mathcal{N} = 1$ supergravity multiplet, and three neutral chiral multiplets associated

[14] Notice that solving for some Kahler parameters does not imply they are stabilized. As discussed in [19], it only implies that for other choices of Kahler moduli the ansatz for the D-brane configuration is non-supersymmetric, and decays into a stable and supersymmetric one. In the T-dual picture it corresponds to recombining intersecting D-branes; in the magnetised D-brane picture it corresponds to transitions to configurations where the gauge bundle involves the non-abelian pieces.

[15] More explicitly, starting from a configuration which is stable at some value of the moduli, we extrapolate it to large volume (keeping the topological classes of each D-brane fixed, hence allowing for unstable branes in the large volume limit). There the chiral spectrum is computed, and is extrapolated back to the stable point in moduli space. Consideration of unstable configurations at intermediate steps is not problematic, since we are interested in topological quantities.

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to the Kahler parameters of the three two-tori. On top of these massless states, we have a tower of states with masses induced by the NSNS and RR fluxes, of order α'/R^3 for large volume (much lighter than the KK replicas).

We are more interested in determining the spectrum on the new sector, the D-branes, and whether it is modified by the NSNS and RR fluxes. Since the fluxes are 'color blind', in the sense that they can couple only to traces over the gauge indices, each stack of N_a D9$_a$-branes still leads to a U(N_a) gauge symmetry. By supersymmetry we obtain 4d $\mathcal{N}=1$ vector multiplets with gauge group $\prod_a \mathrm{U}(N_a)$, just like in the absence of bulk fluxes. This large volume result is presumably robust enough to be maintained at small volumes.

There is however a non-trivial effect from the fluxes. The usual three adjoint chiral multiplets in the aa open string sector are in general expected not to be present, due to interactions with the closed sector, which is only $\mathcal{N}=1$ supersymmetric. The masses for these chiral multiplets seem not to arise at leading order in α' [5], but are certainly expected in the full theory. This effect has the additional advantage of removing the corresponding moduli fields in the open string sector. We also expect this to hold for any non-chiral set of fields not protected by supersymmetry, chirality or gauge symmetry.

In ab sectors (or similarly ab' or aa' sectors), the massless modes are given by $\mathcal{N}=1$ chiral multiplets transforming as bifundamental representations of the corresponding gauge groups. These multiplets arise from zero modes of the Dirac operator coupled to the D9-brane world-volume magnetic field, and are localized at points in the internal space. This implies that in the large volume limit these fields are insensitive to the presence of NSNS and RR field strengths, since their density is small at the location of the zero mode. That is, the turning on of fluxes can be regarded as an adiabatic change in the local region around the zero modes, hence the index of the Dirac operator is unchanged by them. The massless spectrum is therefore still given by the same formulas as in the situation without fluxes. By chirality, the same result holds in the small volume regime. Note that the spectrum of massive excitations (higher Landau levels) will in general be modified by the bulk fluxes.

The main conclusion is that the spectrum of gauge multiplets and chiral matter is still given by the above formulae in table 1 for our models in $\mathbf{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$.[16] We will find additional support for this coming from the analysis of anomalies. Masses of vector-like fields will in general change, and it would be interesting to have an estimate for them.

5.3 Anomaly cancellation

Cubic non-abelian anomalies. There is a close relation between cancellation of RR tadpoles and cancellation of anomalies. In models with NSNS and RR fluxes, where there is an additional contribution to the RR tadpole conditions, it is expected that the usual anomaly cancellation pattern is different. This issue was analyzed in [33], where it was determined that the presence of NSNS and RR fluxes induce explicitly non-gauge invariant Wess-Zumino terms in the action of D-branes.

[16] In non-susy cases of toroidal orientifolds with fluxes and D-branes, we also expect the spectrum to be of the familiar form (reviewed in section 4.1) to be valid at large volume, and by chirality at smaller volume for gauge fields and chiral fermions.

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In our above setup, the coupling on the volume of the D9$_a$-brane stack relevant for the discussion of 4d anomalies is of the form

$$\int_{D9_a} B_{NS} \wedge F_3 \wedge \left(\mathrm{tr}\, F_a^3\right)^{(0)} \tag{5.13}$$

where we are using Wess-Zumino descent notation.[17] The variation of this term under a SU(N_a) gauge transformation induces a 4d anomaly given by

$$\begin{aligned}\int_{D9_a} B_{NS} \wedge F_3 \wedge \delta\left(\mathrm{tr}\, F_a^3\right)^{(0)} &= \int_{D9_a} B_{NS} \wedge F_3 \wedge d\left(\mathrm{tr}\, F_a^3\right)^{(1)} \\ &= \int_{D9_a} H_3 \wedge F_3 \wedge \left(\mathrm{tr}\, F_a^3\right)^{(1)} \\ &= m_a^1 m_a^2 m_a^3\, N_{\mathrm{flux}} \int_{M_4} \left(\mathrm{tr}\, F_a^3\right)^{(1)} \\ &= [Q_a] \cdot [Q_{\mathrm{flux}}] \int_{M_4} \left(\mathrm{tr}\, F_a^3\right)^{(1)} .\end{aligned}$$

Further quotient by the orientifold action cut this contribution by one half.

The total SU($N_a/2$)^3 anomaly is given by this contribution plus the familiar field theory triangle diagrams where dynamical fermions in the spectrum run in a loop. Their net contribution is given by

$$\begin{aligned}&\sum_{b\neq a} I_{ab} \frac{N_b}{2} + \sum_{b'\neq a'} I_{ab'} \frac{N_b}{2} + n_{\square\,\square} \left(\frac{N_a}{2-4}\right) + n_{\square\square} \left(\frac{N_a}{2+4}\right) = \\ &\qquad = [Q_a] \cdot \frac{1}{2} \left(\sum_{\forall b} N_b [Q_b] + \sum_{\forall b} N_b [Q_{b'}] - 32 [Q_{Op}]\right).\end{aligned} \tag{5.14}$$

Thus the total contribution is proportional to $[Q_a] \cdot [Q_{\mathrm{tot.}}]$ which vanishes due to the RR tadpole condition (5.11).

It is easy to check that other anomalies, like mixed U(1)-SU(N_a)^2 or mixed gravitational anomalies also cancel, involving in addition a Green-Schwarz mechanism identical to that in models without NSNS and RR fluxes [15, 19].

Brane-bulk mixed anomalies. In the presence of fluxes the 10d supergravity Chern-Simons interaction (2.3) can lead to the mixing of 4d 1-form U(1) gauge fields (from the KK reduction of the RR-form along a 3-cycle) with 4d scalar (4d dual to the NSNS or RR 2-form) [34]. Indeed this kind of coupling is responsible for giving masses to 4d gauge bosons which are superpartners of the stabilized moduli [4, 5].

On the other hand, this kind of $B \wedge F$ couplings can lead to U(1)-SU(N_a)^2 mixed anomalies between a closed string U(1) gauge field and the SU(N_a) gauge bosons on D9-brane world-volumes, by a Green-Schwarz type diagram. Such anomalies would be however canceled by flux-induced Wess-Zumino terms in the D9-brane world-volumes [34].

[17] Namely for any closed gauge invariant form Y we define $Y = dY^{(0)}$ and $\delta Y^{(0)} = dY^{(1)}$, with δ denoting gauge variation.

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This phenomenon, although present in general models with field strength fluxes, is absent in our configurations. This is because the closed string sector does not contain any gauge bosons; equivalently, because e.g. components of the RR 4-form with three indices in the internal directions are odd under the ΩR projection. Hence our models do not have Green-Schwarz diagrams contribution to mixed anomalies involving bulk and brane gauge interactions.

5.4 Examples

A simple chiral model. Let us start by considering a simple and illustrative example of a chiral model. Consider the flux

$$G_3 = 4 \times \frac{2}{\sqrt{3}}\, e^{-\pi i/6}\, (d\overline{z}_1 dz_2 dz_3 + dz_1 d\overline{z}_2 dz_3 + dz_1 dz_2 d\overline{z}_3) \tag{5.15}$$

which corresponds to the particular example of fluxes (4.18) in [5]. The flux stabilizes the complex structure and complex coupling moduli at values

$$\tau_1 = \tau_2 = \tau_3 = \phi = e^{2\pi i/3}\,. \tag{5.16}$$

The flux is supersymmetric, and it is easy to check that its integral over any 3-cycle of $\mathbf{T}^6$ is multiple of 8

The contribution of the flux to the 4-form RR tadpole (5.8) is

$$N_{\rm flux} = 192\,. \tag{5.17}$$

We can satisfy the RR tadpole conditions (5.11) with the set of magnetised D9-branes in table 2 (and their ΩR images). The main part of the model (except for the 180 $\overline{D}3$-branes in the last line) is supersymmetric for $\arctan A_1 + \arctan A_2 - \arctan A_3 = 0$

The D9-branes along classes invariant under ΩR (refered to as filler branes) lead with vanishing Wilson lines to a maximally enhanced gauge group $\mathrm{USp}(6) \times \mathrm{USp}(6) \times \mathrm{USp}(26)$, and $\mathcal{N} = 1$ chiral multiplets in three copies of the two-index antisymmetric representation of the corresponding symplectic factor. We also get U(1) factors for the $D3$-branes, taken non-coincident. The remaining stack of 10 D9-branes (referred to as GUT branes), and their ΩR images, lead to a U(5) $\mathcal{N} = 1$ vector multiplets and three adjoint chiral multiplets. Additional non-chiral matter arises from strings stretching among the filler branes. It is expected that α' effects generate tree-level masses for all these non-chiral matter multiplets. The interesting chiral matter arises from open strings stretching between the GUT branes and their images, and between the GUT branes and the filler branes. In total, we have the matter content

N_a	(m_a^1, n_a^1)	(m_a^2, n_a^2)	(m_a^3, n_a^3)
10	$(1,1)$	$(1,1)$	$(-1,1)$
6	$(0,1)$	$(1,0)$	$(-1,0)$
6	$(1,0)$	$(0,1)$	$(-1,0)$
26	$(1,0)$	$(-1,0)$	$(0,1)$
90	$(0,-1)$	$(0,-1)$	$(0,-1)$

Table 2: D9-brane configuration for the SU(5) GUT model.

$$\begin{gathered}\mathrm{U}(5) \times \mathrm{USp}(6) \times \mathrm{USp}(6)' \times \mathrm{USp}(26) \\ 8(10;1,1,1) + (5;6,1,1) + (5;1,6,1) + (\overline{5};1,1,26) + 90(\overline{5};1,1,1)\end{gathered} \tag{5.18}$$

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The spectrum is anomalous, but the anomaly is exactly cancelled by the flux induced 4d Wess-Zumino terms, which can be checked to contribute as 96 chiral fermions in the fundamental representation of U(5).

The above example contains a subsector providing a peculiar kind of SU(5) grand unified theory, with 8 families. It is peculiar in that it contains additional fundamental representations unmatched with antifundamentals, so that Dirac mass terms are in principle not possible to get rid of the additional chiral matter, since its anomaly is cancelled via Wess-Zumino terms.

As discussed in section B.2, the model contains some additional D-branes, which are required for consistency of the stacks of D9-branes wrapped on cycles with non-zero H_3. Their presence, however, does not introduce any additional interesting features in the model (at least, concerning its supersymmetry and its chiral spectrum).

Another more interesting chiral model. The existence of Wess-Zumino terms for gauge factors in the would-be Standard Model is not in principle phenomenologically desirable. After all, the gauge representations of standard model fermion *are* anomaly-free. This is not a general obstacle for model building with D-branes in the presence of fluxes. Indeed it is possible to construct models where the would-be Standard Model sector has no Wess-Zumino terms, because the corresponding D-branes have classes with zero intersection with the class of the flux. In this kind of models, the visible sector also does not intersect the $\overline{D}3$-branes, and is hence fully supersymmetric, with supersymmetry broken in a hidden sector.

Consider the same flux configuration as above, but now let us satisfy RR tadpole cancellation using the stacks of D-branes in table 3, plus 180 $\overline{D}3$-branes. The visible sector of this configuration is supersymmetric for $A_1 = 2A_2 = A_3$. Notice that since every stack of branes has at least some $m_a^i = 0$, they are really D7-branes wrapped on two complex planes, with non-trivial worldvolume magnetic fields along them.

N_a	(m_a^1, n_a^1)	(m_a^2, n_a^2)	(m_a^3, n_a^3)
6	$(1,1)$	$(-2,1)$	$(0,1)$
4	$(0,1)$	$(2,1)$	$(-1,1)$
4	$(1,0)$	$(-1,0)$	$(0,1)$
16	$(1,0)$	$(0,1)$	$(-1,0)$
8	$(0,1)$	$(1,0)$	$(-1,0)$

Table 3: D-brane configuration for the four-family model.

The resulting spectrum of vector and chiral multiplets is (at the chiral level)

$$\begin{aligned}
&\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{USp}(4) \times \mathrm{USp}(16) \times \mathrm{USp}(8) \\
&4(3,\overline{2};1,1,1)_{1,-1} + 2(\overline{3},1;1,16,1)_{-1,0} + (3,1;1,1,8)_{1,0} + (1,\overline{2};4,1,1)_{0,-1} + \\
&+2(1,2;1,16,1)_{0,1} + 2(3,1;1,1,1)_{-2,0} + 2(6,1;1,1,1)_{2,0} + 2(1,1;1,1,1)_{0,2} + \\
&+2(1,3;1,1,1)_{0,-2}
\end{aligned}$$

The model is very close to a four-family Standard Model (in fact, appeared in [19], with some branes replaced by fluxes). However, it lacks a massless candidate U(1) for hypercharge, since the suitable linear combination become massive due to $B \wedge F$ couplings [35]. Hence the model is admittedly not realistic, but it provides a good illustration of model building possibilities. Notice that the supersymmetric visible sector is really decoupled (modulo closed string interactions) from the supersymmetry breaking hidden sector. Note

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also that the latter is relatively stable, since annihilation processes between fluxes and branes are quite suppressed [47], and is in fact very similar to recent proposal to build de Sitter vacua in string theory with large flux quanta and $\overline{D}3$-branes [10].

It is not our purpose to start a systematic exploration of model building in this setup, which we leave for future work. We would like to emphasize however that much more freedom is expected if one considers the possibility of taking flat directions in initial supersymmetric models with larger gauge group. This has been successfully exploited in other supersymmetric models in the literature. We leave this avenue for richer model building for future work.

6. Discrete B field

The configurations we are considering admit a discrete modification, corresponding to including a discrete value of the NSNS 2-form field along some of the two-tori [16]. This possibility is interesting as a possible way of obtaining models with odd number of fermion families (although not the only one, if $SU(2)_L$ is embedded as USp(2) in some filler branes [36]). In this section we describe the main modifications introduced by non-trivial B-fluxes, centering for simplicity on the case of B-flux on a single two torus, say the third.

A first modification [16] is that D9-branes carry induced lower-dimensional brane charges due to the 1/2 unit of B_{NSNS}. It is then useful to introduce an effective world-volume magnetic flux quantum, defined by $\tilde{n}^3_a = n^3_a + \frac{1}{2}m^3_a$.

A second effect is that the discrete B-field changes the RR tadpole cancellation conditions, decreasing by a factor of two the total charge associated to O-planes pointlike in the corresponding two-torus. The physical interpretation of this has been discussed quite explicitly in section 3 of [37]: 1/4 of the corresponding O-planes carry positive RR charge, while 3/4 of them are the familiar negatively charged O-planes. The net effect is to cut by half the total contribution of the O-planes to the RR charge, as compared to the zero B-field case. Hence in our setup, the model contains 48 $O3^-$-planes and 16 $O3^+$-planes, and 3 $O7_3^-$-planes and one $O7_3^+$-plane.

Gathering the two results together, the RR tadpole conditions read

$$\begin{aligned}
\sum_a N_a n^1_a n^2_a \tilde{n}^3_a + \frac{1}{2}N_{\rm flux} - 8 &= 0 \\
\sum_a N_a n^1_a m^2_a m^3_a + 16 &= 0 \\
\sum_a N_a m^1_a n^2_a m^3_a + 16 &= 0 \\
\sum_a N_a m^1_a m^2_a \tilde{n}^3_a + 8 &= 0\,.
\end{aligned} \tag{6.1}$$

Finally, there might be a subtle effect on the 3-form fluxes we are allowed to turn on. As noticed in [4, 5] for the toroidal orientifold case, the quantization conditions of 3-form fluxes depend on the kind of O3-planes in the configuration. There are four kinds of O3-planes, classified [8] by the $\mathbb{Z}_2$ value (r_1, r_2) of the NSNS and RR 2-forms on an $\mathbf{RP}_2$ surrounding the O3-plane in the transverse 6d space. For instance, the $O3^-$-, $O3^+$-

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planes discussed above correspond to O3-planes with $(r_1, r_2) = (0,0)$, $(1/2,0)$, resp. If the integral of the NSNS (resp. RR) 3-form field strength along one of the basis 3-cycles is integer but odd, consistency requires the corresponding 3-cycle to pass through an odd number of O3-planes with $r_1 = 1/2$ (resp. $r_2 = 1/2$).

We would like to provide an alternative derivation of this rule. For each basic 3-cycle C in $\mathbf{T}^6$, there is a 3-cycle $\tilde{C}$ closed on $\mathbf{T}^6/(\Omega R)$ which is not closed on $\mathbf{T}^6$ and has half its volume. For instance, the 3-cycles given by $\mathrm{Re}z_1 \in [0,1)$, $\mathrm{Re}z_2 \in [0,1)$, $\mathrm{Re}z_3 \in [0,1/2)$. If e.g. $\int_C H_3 =$ odd, then $\int_{\tilde{C}} H_3 = 1/2 \bmod \mathbb{Z}$. We would like to correlate this with the properties of the O3-planes through which $\tilde{C}$ and C pass. To do this, remove a small 3-ball in $\tilde{C}$ around each O3-plane, whose boundary is an $\mathbf{RP}_2$, due to the orientifold quotient. We obtain a submanifold $\tilde{C}'$ with boundary, $\partial\tilde{C}'$ homologically given by the sum of the classes of the $\mathbf{RP}_2$'s, denoted Σ_i. The H_3-flux $\int_{\tilde{C}'} H_3$ is however still equal to $\int_{\tilde{C}} H_3 = 1/2 \bmod \mathbb{Z}$. From these conditions we have

$$\int_{\tilde{C}'} H_3 = \int_{\partial\tilde{C}'} B_2 = \sum_i \int_{\Sigma_i} B_i = \sum_i (r_1)_i \tag{6.2}$$

Hence $\sum_i (r_1)_i = 1/2 \bmod \mathbb{Z}$, which implies that the 3-cycle passes through an odd number of O3-planes with $r_1 = 1/2$.

One may expect that in our configurations, the presence of O3$^+$-planes leads to modified quantization conditions. However, it is easy to verify that in the simplest case of non-zero B-field in just one two-torus, the configuration of O3-planes is such that any 3-plane passes through an even number of O3$^+$-planes. Therefore the correct quantization conditions require even H_3 and F_3 flux integrals over the basic 3-cycles.[18] Configurations with B-fields in several complex planes may however introduce modified quantization conditions.

N_a	(m_a^1, n_a^1)	(m_a^2, n_a^2)	$(m_a^3, \tilde{n}_a^3)$
8	$(-1,1)$	$(1,0)$	$(-2,-1)$
4	$(-1,0)$	$(1,-1)$	$(2,1)$
2	$(0,1)$	$(0,1)$	$(0,1)$
12	$(1,0)$	$(-1,0)$	$(0,1)$
12	$(1,0)$	$(0,1)$	$(-2,0)$

Table 4: D9-brane configuration for the model with B-flux.

Let us provide one example of model with non-trivial B-field, just for illustration. Consider the D-brane configuration in table 4.

And let us saturate the RR tadpole with a flux $N_{\rm flux} = 192$ and 180 $\overline{D}3-branes$ like the above. Supersymmetry in the remaining branes is preserved for suitable choices of the compactification areas. The chiral spectrum is

$$\begin{aligned}
&\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{USp}(2) \times \mathrm{USp}(12) \times \mathrm{USp}(12) \\
&4(4,2;1,1,1)_{1,1} + 2\left(\overline{4},1;2,1,1\right)_{-1,0} + 2\left(\overline{4},1;1,1,12\right)_{-1,0} + 4(6,1;1,1,1)_{-2,0} + \\
&+4(10,1;1,1,1)_{2,0} + 2(1,2;2,1,1)_{0,1} + 4(1,1;1,1,1)_{0,2} + 4(1,3;1,1,1)_{0,-2} + \\
&+180(4,1;1,1,1)_{1,0} + 180(1,2;1,1,1)_{0,1}\,.
\end{aligned} \tag{6.3}$$

[18] One may worry about a possible correlation between such quanta and the RR charge of the O7-planes. However, the above geometric argument to show this correlation does not work, since the 3-cycles necessarily are not transverse to the O7-plane, and the boundaries of the small 3-balls do not correspond to $\mathbf{RP}^2$'s located at the O7-planes. Hence O7-planes seemingly do not modify the quantization conditions imposed by the O3-plane distribution.

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Anomalies cancel using the additional contribution from the WZ terms. The model has a Pati-Salam gauge group (which can be subsequently broken to the Standard Model with correct hypercharge, by separating branes). However, the structure of families is not satisfactory, due to the contribution of the flux-induced WZ terms. In any event, it provides a simple example of model building in this setup.

7. T-dual version and D-branes on non-Calabi-Yau manifolds

Models with NSNS and RR fluxes transform under T-duality in an interesting way. Denoting by x the T-duality direction, the rough rules are as follows: i) RR forms transform by increasing or decreasing their degree for components transverse to or along x, respectively; ii) components of the NSNS 3-form field H_3 transverse to x remain invariant; iii) finally, components of H_3 along x (and two additional directions y, z) transform into components of metric curvature in the T-dual geometry, which forms a non-trivial bundle of the T-dual circle over the subspace parametrized by the coordinates y, z. The first Chern class of this bundle is given by the total flux of the original H_3 along x, y, z.

The effect of T-duality has been studied quite generally in [23], and in the particular case of the torus in [25]. It has been argued that geometries related by T-duality to Calabi-Yau compactifications with fluxes are neither Kahler nor complex, and hence not Calabi-Yau. In supersymmetric cases they however admit a (non-Levi-Civita) connection of SU(3) holonomy. As discussed in [23], the geometry can be described (in a particular limit) as a deformation of a Calabi-Yau space by a metric deformation characterized by some forms, which we refer to as 'metric fluxes'. The study of general compactifications with metric fluxes (as well as other fluxes) seems a promising direction towards understanding more general string compactifications.[19]

From this viewpoint, our models in this paper, upon application of T-duality, can be turned into the first (almost) supersymmetric chiral compactifications on non-Calabi-Yau manifolds with D-branes. These T-dual models are simpler to describe as deformations by metric fluxes of the T-dual toroidal orbifolds. Specifically, perform a T-duality along the horizontal direction x^8 in the third two-torus, denoted x for shorthand. The dual model corresponds to type-IIA compactified on a (metric flux deformed, see later) $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold, modded out by $\Omega R'$, with $R' : x^a \to -x^a$, $a = 4, 5, 6, 7, 9$, $R : x \to x$, where the T-dual coordinate is also denoted x. Needless to say, our discussion applies even more simply to T-duality for toroidal orientifolds with fluxes, although in this case the complete configurations are non-supersymmetric.

The T-dual background includes RR 2- and 4-form field strength background, and NSNS 3-form background. In addition, components $(H_3)_{abx}$, introduce a non-trivial metric component

$$g_{bx} \propto x^a \tag{7.1}$$

in the T-dual space, in fact turning the corresponding 3-torus (spanned by x^a, x^b, x) into a non-trivial $\mathbf{S}^1$ bundle (with fiber parametrized by x) over the 2-torus spanned by x^a, x^b.

[19] See [23, 24, 25] for other recent work on this kind of compactification, and e.g. [26] for applications to heterotic compactification.

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Although the space is topologically different, locally on the base the geometry differs from a toroidal one by the 1-form

$$g_{(x)} = \frac{g_{ax}}{g_{xx}} dx^a \tag{7.2}$$

which encodes the twisting [25]. Its curvature $\omega_{(x)} = -dg_{(x)}$ is the curvature of the non-trivial bundle.

The above orientifold quotient introduces a number of O4-planes, spanning M_4 and the direction x. Part of the corresponding RR tadpole is saturated by the fluxes. The remaining is saturated by D-branes, which in our supersymmetric case are D8-branes wrapped on the first two two-tori, times a 1-cycles in the third one. Regarding the T-dual background as a toroidal orbifold deformed by a metric flux, which is the right viewpoint from the results in [23], allows to discuss the D-brane configurations quite explicitly, almost as simply as in the absence of field strength fluxes.

In the following we would like to discuss two important topological couplings which have not been mentioned in detail in the literature, although they play a crucial role in the construction of compactifications with metric fluxes, and the introduction of D-branes in them. In particular, we describe that combinations of metric fluxes and RR fluxes contribute to RR tadpoles, and to Wess-Zumino terms in D-brane worldvolumes. Although motivated by our specific orbifold models, our analysis in this section is completely general and valid for other compactifications.

7.1 RR tadpoles and metric fluxes

T-duality implies that certain combinations of RR and metric fluxes are sources for certain RR potentials. Hence type-IIA compactified on a non-Calabi-Yau manifold $\mathbf{X}_6$ corresponding to Calabi-Yau space $\mathbf{Y}_6$ deformed with metric fluxes encoded in a 1-form $g_{(x)}$ must contain Chern-Simons couplings responsible for this phenomenon. In this section we describe how they arise.

From the T-dualization of the type-IIB coupling (2.3) along a component of H_3, the couplings we need are

$$\int_{10d} g_{(x)} \wedge F_{4(x)} \wedge F_6 = \int_{10d} \omega_{(x)} \wedge F_{4(x)} \wedge C_5 \tag{7.3}$$

where we have introduced the notation $(F_p)_{(x)} = F^{(p)}_{a_1 \dots a_{p-1} x} dx^{a_1} \dots dx^{a_{p-1}}$. This coupling shows that RR 4-form and metric fluxes generate a tadpole for the RR 5-form potential.

We now show that this coupling simply arises from the expasion of the kinetic term for the RR 3-form on $\mathbf{X}_6$, with metric g, around the Calabi-Yau metric g_0, to first order in the metric flux deformation δg. We have the structure

$$g_0 = \begin{pmatrix} g_{xx} & 0 \\ 0 & g_{ab} \end{pmatrix}; \qquad \delta g = \begin{pmatrix} 0 & g_{ax} \\ g_{ax} & 0 \end{pmatrix} \tag{7.4}$$

and $g = g_0 + \delta g$. Expanding a piece of kinetic term of the RR 5-form on $\mathbf{X}_6$ around g_0 to first order in δg, we have

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$$
\begin{aligned}
S_{\rm kin.g} &= \int_{M_4\times \mathbf{X}_6} F_{6(x)} \wedge *_g F_{6(x)} \\
&= \int_{M_4\times \mathbf{X}_6} \sqrt{g}\, F_{0123xa}\, F^{0123xa}\, d(\mathrm{Vol}) \\
&= \int_{M_4\times \mathbf{X}_6} \sqrt{g}\, F_{0123xa}\, F_{0123\mu\nu}\, g^{\mu x}\, g^{\nu a}\, d(\mathrm{Vol})
\end{aligned}
\tag{7.5}
$$

where $\mu,\nu = 4,\ldots,9$. The zeroth order in δg leads to the kinetic term of the 5-form in the background metric g_0 of $\mathbf{Y}_6$. The first order in δg is obtained by taking $\sqrt{g} \to \sqrt{g_0}$, $\mu,\nu \neq x$. We obtain

$$
\Delta S = \int_{M_4\times \mathbf{Y}_6} \sqrt{g_0}\, F_{0123xa} F_{0123bc}\, g^{bx}\, g^{ca}\, d(\mathrm{Vol})
\tag{7.6}
$$

We now use the Hodge duality relation in $\mathbf{Y}_6$, $F_6 = *_{g_0} F_4$. Replacing

$$
F_{0123bc} = \sqrt{g_0}\, \epsilon_{0123bca_1a_2a_3x}\, (g_0)^{a_1b_1}\, (g_0)^{a_2b_2}\, (g_0)^{a_3b_3}\, (g_0)^{xx}\, F_{b_1b_2b_3x}
\tag{7.7}
$$

into (7.6), and using $g^{bx} = -(g_0)^{bd} g_{dx} (g_0)^{xx}$, we get

$$
\begin{aligned}
\Delta S &= -\int_{M_4\times \mathbf{Y}_6} (\det g_0)\, F_{0123xa}\, F_{b_1b_2b_3x}\, \big(g_{(x)}\big)_d\, \epsilon^{0123dab_1b_2b_3x}\, d(\mathrm{Vol}) \\
&= \int_{M_4\times \mathbf{X}_6} g_{(x)} \wedge F_{4(x)} \wedge F_6\,.
\end{aligned}
\tag{7.8}
$$

This generalizes to similar couplings for other RR forms. The general conclusion is that the kinetic term for a RR form in a geometry deformed by metric fluxes leads to the kinetic term in the undeformed metric plus a Chern-Simons coupling of the metric flux to RR fields.

7.2 World-volume WZ terms and metric fluxes

A similar analysis can be carried out for other important couplings in the presence of metric fluxes. For instance, the existence of world-volume WZ couplings for D9-branes in the presence of H_3, F_3 fluxes [33]

$$
\int_{D9} B_2 \wedge F_3 \wedge \big(\mathrm{tr}\, F^3\big)^{(0)} = \int_{D9} B_2 \wedge C_2 \wedge \mathrm{tr}\, F^3
\tag{7.9}
$$

implies that D8-branes in the presence of RR 4-form and metric fluxes develop a WZ term

$$
\int_{D8} g_{(x)} \wedge F_{4(x)} \wedge \big(\mathrm{tr}\, F^3\big)^{(0)} = \int_{D8} g_{(x)} \wedge C_{3(x)} \wedge \mathrm{tr}\, F^3\,.
\tag{7.10}
$$

Such terms indeed arise on D8-branes in the presence of metric fluxes. In fact, they come from the Chern-Simons terms of D8-branes in $\mathbf{X}_6$,

$$
\int_{D8} P(C_3) \wedge \mathrm{tr}\, F^3
\tag{7.11}
$$

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expanded around the configuration in $\mathbf{Y}_6$, taking into account that the pullback of C_3 has an expansion

$$P(C_3)_{abc} = (C_3)_{abc} + \partial_c x\,(C_3)_{abx}\,. \tag{7.12}$$

Here x denotes the embedding of the D8-brane in the non-trivial $\mathbf{S}^1$ bundle, and hence $\partial_a x = (g_{(x)})_a$. Thus the first term in the expansion gives

$$\int_{D8} g_{(x)} \wedge C_{3(x)} \wedge \mathrm{tr}\, F^3 \tag{7.13}$$

explaining the required coupling.

Using these arguments it is straightforward to translate any model constructed in terms of H_3, F_3 fluxes into a model with non-trivial metric fluxes, at least for T-duality along a single direction. We expect much research in the construction of D-brane configurations in these non-Calabi-Yau spaces.

8. Final comments

Compactifications with field strength fluxes are a most promising avenue for model building, in that they provide a canonical mechanism to stabilize most moduli of the compactification.

In this paper we have studied the construction of compactifications with D-branes, leading to chiral gauge sectors, and moduli stabilization by NSNS and RR fluxes. We have described different approaches to achieve this aim, and provided explicit examples with D3-branes at singularities (with supersymmetry broken in the closed string sector) and D-branes with world-volume magnetic fluxes (with a supersymmetric visible sector and a supersymmetry breaking hidden one). Along the way we have covered some new interesting properties of the models, like how to address the subtle behaviour of D-branes in the presence of NSNS fluxes, the relation between tadpoles and anomalies, etc.

Although we mainly centered in (almost) supersymmetric model building, our techniques can be inmediatly applied to the construction of non-supersymmetric models, for instance in toroidal orientifolds. We expect that within this less restrictive setup one can achieve the construction of far more phenomenologically appealing gauge sectors.

Many further directions remain open. In the context of moduli stabilization, it would be interesting to explore the interplay between the flux induced scalar potential with other sources of potential for moduli, like non-supersymmetric sets of D-branes, or non-perturbative corrections. This step is crucial in order to understand the fate of the moduli which are not stabilized by the fluxes. It would also be interesting to determine patterns for the values at which moduli stabilize, in order to understand for instance what properties the underlying model must have in order to lead to e.g. small 4d gauge couplings, or large radii.

In the context of generalizing these constructions, a nice extension of our models would be the construction of compactifications with fluxes and chiral D-branes in more general Calabi-Yau spaces. Also, as we have discussed, compactification with fluxes are closely related to compactifications on non-Calabi-Yau manifolds, related to metric fluxes. Study of the latter is one of the most exciting directions in generalizing our knowledge of string

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theory vacua. Introduction of D-branes in them would largely enhance our model building possibilities, bringing us perhaps one step closer to understanding the structure of the observed physics.

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A. NSNS and RR fluxes in a type-IIB orientifold with D3-branes at singularities

In this appendix we would like to discuss the construction of orientifold of type-IIB on $\mathbf{T}^6/\mathbb{Z}_3$ by the orientifold action ΩR with $R : z_i \to -z_i$, and NSNS and RR 3-form field strength fluxes. This is a T-dual version of the model in [38]. As discussed in the main text, the models turn out be non-supersymmetric in the closed string sector since the orbifold and the $\mathbb{Z}_3$ invariant fluxes necessarily preserve different supersymmetries.

A direct way of constructing would be to introduce fluxes in the $\mathbf{T}^6/\mathbb{Z}_3$ orientifold, which will necessarily be non-supersymmetric, but should at least be imaginary self-dual with respect to the metric of the 6d internal space.

We prefer instead to take an indirect route, which takes more advantage of our discussions in the main text, and carry out the construction as follows. We start with type-IIB on $\mathbf{T}^6/\mathbb{Z}_3$ modded out by ΩR, and introduce a 3-form flux of the form

$$G_3 = 2\, dz_1 dz_2 d\overline{z}_3 \,. \tag{A.1}$$

This flux stabilizes moduli at a factorized product of three two-tori (i.e. off-diagonal Kahler parameters are frozen to zero) with complex structure parameters $\tau_i = e^{2\pi i/3}$, and stabilizes the dilaton at $\phi = e^{2\pi i/3}$. It is also properly quantized over 3-cycles in the torus, with integrated fluxes giving even numbers. Notice that this flux preserves some of the supersymmetries of the underlying $\mathbf{T}^6/(\Omega R)$ geometry. Concretely it preserves the spinor ξ_0 (satisfying $\gamma^{\overline{i}}\xi_0 = 0$ in the above complex coordinates), as well as the spinors $\gamma^1\gamma^3\xi_0$ and $\gamma^2\gamma^3\xi_0$ (hence preserves 4d $\mathcal{N} = 3$ supersymmetry). Being supersymmetric the flux is automatically imaginary self-dual with respect to the underlying metric. For the above flux we have $N_{\rm flux} = 12$, hence cancellation of RR tadpoles requires the introduction of 20 D3-branes.

We now mod out the configuration by the $\mathbb{Z}_3$ orbifold generated by

$$\theta : (z_1, z_2, z_3) \to \left(e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{4\pi i/3} z_3\right) \tag{A.2}$$

namely $z_i \to e^{2\pi v_i} z_i$ with $v = 1/3(1, 1, 2)$.

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For this quotient to be possible it is crucial that the flux (A.1) is invariant under the action of θ, so it corresponds to a possible flux in $\mathbf{T}^6/\mathbb{Z}_3$ (fulfills condition i) in section 2.3). It is also important to notice that the $\mathbb{Z}_3$ quotient of $\mathbf{T}^6/(\Omega R)$ does not contain closed 3-cycles which are not closed in $\mathbf{T}^6/\mathbb{Z}_3$. hence proper quantization is not spoilt (point ii in section 2.3). Also, the collapsed cycles at $\mathbb{Z}_3$ singularities are 2- and 4-cycles, hence do not impose additional quantization constraints (point iii in 2.3). Finally, the flux is imaginary self-dual in the metric of the orbifold space (at the orbifold point in moduli space), since the metric is inherited from that of $\mathbf{T}^6$ (point iv in 2.3). Finally, the spinor preserved by the orbifold projection is $\gamma^3\xi^0$, which is not any of the supersymmetries preserved by the flux. Hence the final model is not supersymmetric.

In this model, supersymmetry broken in the closed string sector by the interplay between the orbifold projection and the flux. It is possible that this kind of breaking of supersymmetry has some particularly nice features, since the interactions between untwisted modes is sensitive to supersymmetry breaking only via effects involving twisted modes. It would be interesting to analyze the impact of this property on the violations of the no-scale structure of the low energy supergravity effective theory for these models (i.e. the degree of protection against α' or g_s corrections).

To define the model completely, we need to specify the configuration of the 20 D3-branes, which are still required to cancel the untwisted RR tadpole. In the ΩR orientifold of $\mathbf{T}^6$ there is one point, the origin $(0,0,0)$, fixed under ΩR and θ. At this point, cancellation of RR twisted tadpoles requires the presence of D3-branes, with a Chan-Paton matrix satisfying

$$\mathrm{Tr}\,\gamma_{\theta,3} = -4 \tag{A.3}$$

In addition, there are other 26 points fixed under θ (and gathered in 13 pairs under ΩR), where there is no twisted RR tadpole. If D3-branes are present, they should have traceless Chan-Paton matrix. Finally there are 63 points fixed under ΩR (gathered in 21 trios under $\mathbb{Z}_3$) at which we may locate any number (even or odd) of D3-branes.

A simple solution would be to locate the 20 D3-branes at the origin, with

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathbf{1}_4, e^{2\pi i/3}\mathbf{1}_8, e^{4\pi i/3}\mathbf{1}_8\right) \tag{A.4}$$

leading to an $\mathcal{N}=1$ supersymmetric sector (to leading approximation, since interactions with the closed sector would transmit supersymmetry breaking), with spectrum

$$\begin{aligned} &\mathcal{N}=1 \text{ vect. mult.} \qquad \mathrm{SO}(4)\times\mathrm{U}(8) \\ &\mathcal{N}=1 \text{ ch. mult.} \qquad 3\left[(\square,\square)+\left(1,\overline{\square\,\square}\right)\right]. \end{aligned} \tag{A.5}$$

A more interesting possibility, which we adapt from [39], is to locate 11 D3-branes at the origin, with

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathbf{1}_1, e^{2\pi i/3}\mathbf{1}_5, e^{4\pi i/3}\mathbf{1}_5\right). \tag{A.6}$$

This leads to a gauge sector with

$$\begin{aligned} &\mathcal{N}=1 \text{ vect.mult.} \qquad \mathrm{U}(5) \\ &\mathcal{N}=1 \text{ ch. mult.} \qquad 3\left(5+\overline{10}\right). \end{aligned} \tag{A.7}$$

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One should now be careful in locating the additional D3-branes. Introducing an odd number of D3-branes on top of the O3-plane at the origin implies that it is an $\widetilde{O3}^-$-plane in notation of [8], i.e. there exists a $\mathbb{Z}_2$ B_{RR} background on an $\mathbf{RP}_2$ around the O3-plane. In order to be consistent with the fact that our flux has even integral over the different 3-cycles implies that there should exist other $\widetilde{O3}^-$-planes in the configuration. The conditions in [4, 5] state that for any 3-plane over which the integrated flux of H_3 is even (any 3-plane in our case), the number of $\widetilde{O3}^-$-planes must be even. The configuration in [39] turns out to satisfy the corresponding consistency conditions (the underlying reason being that it is consistent for zero fluxes, which is a particular case of even quanta on 3-cycles).

To adapt this configuration, denote A, B or C the coordinate of an O3-plane in a complex plane, according to whether $z_i = 1/2$, $z_i = e^{2\pi i/3}/2$ or $z_i = (1 + e^{2\pi i/3})/2$. The remaining 9 D3-branes in the model are located on top of O3-planes at the points

$$\begin{array}{lll} (A,A,A) & (B,B,C) & (C,C,B) \\ (A,0,0) & (B,0,0) & (C,0,0) \\ (0,A,0) & (0,B,0) & (0,C,0)\,. \end{array} \tag{A.8}$$

This set is invariant under exchange of fixed points by $\mathbb{Z}_3$, and introduces the right number of $\widetilde{O3}^-$-planes at the right places. The additional D3-branes do not lead to additional gauge symmetries.

Thus the final model contains a 3-family SU(5) GUT gauge sector (although without adjoint chiral multiplets to break it down to the Standard Model), as the only gauge sector of the theory. In addition, its closed string sector is non-supersymmetric. It would be interesting to estimate the impact of the supersymmetry breaking on the gauge sector [40]. It would also be interesting to construct other models based on the $\mathbb{Z}_3$ orbifold, or other orbifold models. We leave these interesting question for future work.

B. K-theory discussion

B.1 Wrapped D-branes with non-zero closed string field strength fluxes

Configurations of D-branes in the presence of NSNS field strength fluxes are subtle. Stable D-brane configurations are characterized by their charges, which are classified by a suitable K-theory group. In simple situations, the K-theory classes are in one-to-one correspondence with cohomological classes, and stable D-branes can be characterized by specifying the homology cycle they wrap (and the Chern classes of the gauge bundle they carry). In other situations, K-theory may differ from cohomology and it may not be possible to wrap a D-brane on a non-trivial homology cycle, or a D-brane in a non-trivial homology cycle may decay and disappear. In the presence of NSNS 3-form field strength it has been proposed that the K-theory group classifying D-brane charges is the twisted K-theory group $K_{[H]}$ of spacetime [30]. This has been argued mainly when the cohomology class $[H_3]$ is torsion [41, 42], and there have been attempts at defining it properly for non-torsion $[H_3]$ [43]. This has proved quite difficult, even conceptually (e.g. definition of a D-brane system as the final state of annihilation of spacetime filling brane-antibrane pairs seems to require an infinite number of such parent pairs).

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On the other hand, an alternative operational definition of the set of allowed D-brane charges in a configuration, including the presence of NSNS field strength was provided in [44], based on a physical understanding of instanton processes mediating D-brane decays (see [45] for a recent discussion). The two basic rules[20] to classify topological D-brane charges on a spacetime $X_9 \times \mathbb{R}$ with NSNS 3-form field strength H_3 are

i) The configuration given by a D-brane wrapping a homologically non-trivial cycle W is consistent, unless the pullback of the bulk 3-form field strength H_3 onto W defines a cohomologically non-trivial class in W.

ii) A configuration of a Dp-brane wrapped on a homologically non-trivial p-cycle W (and propagating in time) is unstable to decay into the vacuum if there exists a $(p+3)$-cycle W', containing W, such that the pullback of H_3 onto W' is the Poincare dual of the class of W in W'.

The decay of the D-branes in ii) is mediated by the following instanton process: Due to i) we cannot wrap a D-brane in W'. If we wrap an euclidean D$(p+2)$-brane on it, the inconsistency can be cured by introducing a Dp-brane wrapped on W (and propagating in time) and ending on it. The D$(p+2)$-brane on W' therefore is an instanton mediating the decay of the Dp-brane on W. The process is shown in figure 1.

The above rules can be explained in more pedestrian terms with explicit examples. To derive i) from physical considerations, take a D3-brane wrapped in a 3-cycle Σ such that $\int_\Sigma H_3 = k$. On the D3-brane worldvolume there is a coupling

$$\int_{\Sigma_3 \times \mathbb{R}} H_3 \wedge \tilde{A}_1 \tag{B.1}$$

where $\tilde{A}_1$ is the gauge potential dual to the world-volume 1-form gauge field. This implies that the H_3 background generates a world-volume tadpole for $\tilde{A}_1$, which renders the theory inconsistent (since it is not possible to satisfy the equations of motion for $\tilde{A}_1$. The inconsistency is avoided if there are k D1-branes which span the direction $\mathbb{R}$ above, and other direction, in which they are of semi-infinite extent because they end on the D3-brane. The dimensions are described by

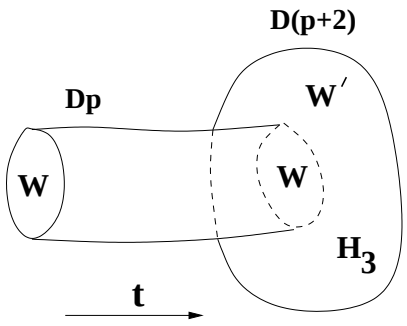

Figure 1: A Dp-brane ending on a D$(p+2)$-brane with volume along some component of H_3. This represents a consistent way of wrapping a D$(p+2)$-brane on a cycles with non-zero H_3 flux, or a process mediating the decay of a Dp-brane wrapped on a homologically non-trivial cycle W.

D3	0	×	×	×	×	×	×	7	8	9
D1	0	×	×	×	×	×	⊣	×	×	×

where ⊣ denotes that the brane is semi-infinite in the corresponding direction. The boundary of the D1-branes on the D3-brane contributes k units of charge under $\tilde{A}_1$, cancelling the world-volume tadpole. Regarding the direction 6 as time, this configuration described above represents the decay of k D1-branes via a D3-brane instanton. The configuration is

[20] We are considering a situation with trivial Stiefel-Witney classes, pertinent to our applications.

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easily T-dualized into a set of Dp-branes spanning $\mathbb{R}^p$ times a semi-infinite line, ending on a D$(p+2)$-brane spanning $\mathbb{R}^p \times \Sigma$. The boundary of the D$p$-branes cancel the contribution from H_3 to the world-volume dual gauge potential $\tilde{A}_p$.

These physically motivated rules, as well as their dual versions, provide an operational definition of conserved D-brane charges which we exploit in the analysis of our configurations.

B.2 D9-branes wrapped on 3-cycles with non-zero H_3

Our configurations in the present paper contain D9-branes wrapped on the internal space, on which we have turned on non-trivial field strength fluxes. Since the pullback of H_3 into the D9-brane volume is non-zero, the configurations are strictly speaking not consistent as they stand. We need to introduce additional D-branes 'ending' on the D9-branes to render the latter consistent. Notice that the fact that our configurations do not contain net D9-brane charge (in a sense contain D9- and anti-D9-branes[21]) does not avoid the inconsistency, which appear for each D9-brane independently.

To understand what additional D-branes we need, consider the simplified situation of D9- anti-D9-brane pairs feeling just one component of H_3, say along 789. T-dualizing along, say 6, this configuration is equivalent to D8 - anti-D8-branes along 012345789 with H_3 along 789. The latter configuration can be made consistent by adding sets of k D6-branes starting from each anti-D8-brane and ending on each D8-brane, as shown in figure 2. T-dualizing back to D9-branes, the required additional D-branes are D5-branes, sitting at a point in the directions 789 where H_3 is non-trivial, and in an additional direction. The fact that the T-dual D6-branes had finite extent and stretched between D8-branes implies the D5-branes must be 'fractional' in the sense of [46].

Hence in our discussion in this paper we assume implicitly that additional branes of this kind have been added to our configurations, to render them consistent.[22] Since our flux configurations are more involved, we do not specify the additional brane content in detail.

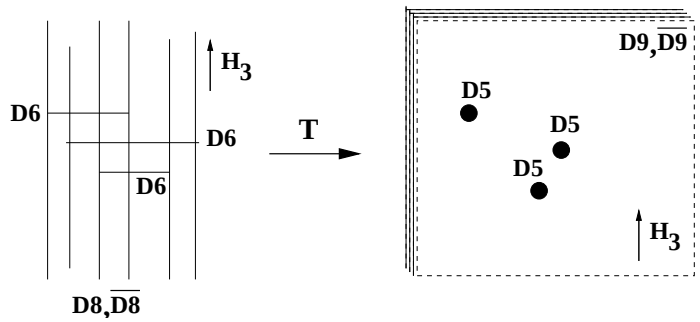

Figure 2: A system D8 - (anti-D8)-branes with volume along some component of H_3 is consistent if there are additional D6-branes starting and ending on them. The T-dual version is that a system of D9 - (anti-D9)-branes with volume along some component of H_3 is consistent if additional fractional D5-branes are present.

[21] In a supersymmetry preserving fashion due to their worldvolume magnetic fields.

[22] The role of the additional branes is even richer. D-branes in the presence of H_3 flux carry in principle not standard gauge bundles, but twisted gauge bundles [30], defined in terms of a cocyle related to the class of H_3. This is analogous to saying that H_3 is magnetically charged under the world-volume gauge

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However, we need not be too specific about it. We would like to emphasize that their presence does not alter any of the main properties of our interest. First, given that the D9-brane content and the fluxes preserve supersymmetry, we expect the additional branes to do so as well. A more precise argument would be as follows: Consider, in the absence of closed string fluxes, a $D9-\overline{D9}$ pair, supersymmetric due to their world-volume gauge bundles. Open strings stretched between them lead to 4d $\mathcal{N}=1$ chiral multiplets, with scalars parametrizing a flat direction along which the $D9-\overline{D9}$-brane form a(n irreducible) bound state, preserving the same supersymmetry as before. Consider now re-introducing the closed string fluxes, preserving that supersymmetry. The D-brane configuration is unchanged, i.e. no additional D-branes are required, since the D-brane bound state carries zero D9-brane charge, and hence does not wrap a 3-cycles with H_3 flux. The D-brane configuration is therefore supersymmetric along the complete flat direction, and so must be at the origin, where the bound state becomes reducible into the configuration of a D9-brane, a $\overline{D9}$-brane, and the additional branes.

Second, it is easy to realize that the additional branes do not introduce additional chiral matter in the spectrum of the theory. This is manifest in the picture of D6-branes ending on D8-branes, where the intersection point is known to preserve 8 supercharges, and lead only to vector-like matter. Therefore the main properties of the models can be obtained ignoring the subtlety of the presence of the additional branes, as we do in the main text[23]

B.3 Instantons violating homological D-brane charge

Another striking feature of D-branes in the presence of H_3 NSNS 3-form field strength is that there exist processes violating conservation of the homological charge carried by the D-branes, via ii).

This phenomenon occurs for D-brane charge associated to classes point-like in the directions spanned by H_3. In our models, the only net homology charges carried by the D-branes are D3-brane charge and D7-brane charge. Only the D3-brane charge is pointlike in the directions spanned by H_3, hence there are processes changing the D3-brane charge. Indeed, for each independent 3-cycle Σ over which $\int_\Sigma H_3 = k$, there exist an 'instanton', given by an euclidean D5-brane wrapped on Σ and spanning the three non-compact spatial dimensions (and at some fixed time t_0), which changes the number of D3-branes by k units.

Since the amount of D3-brane charge was fixed by cancellation of the RR tapdoles generated by the O3-planes, one may wonder how the latter is cancelled after the transition. The answer follows form noticing that the D5-brane instanton couples magnetically to B_{RR},

potential, so the latter is not globally well defined. Because of the additional branes, with magnetic charge cancelling the above, the twisting can be safely ignored (equivalently, gauge fixed to have support at the location of the additional brane), and one can work with usual world-volume bundles. This is essential in order to define magnetised D-branes in the familiar fashion.

[23] The presence of additional branes was unnoticed in the discussion of other configurations of D-branes and fluxes in the literature [33, 34]. Happily their addition does not modify the chiral content and anomaly cancellation mechanisms there discussed.

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hence

$$dF_3 = \delta(D5) \tag{B.2}$$

where $\delta(D5)$ is a bump 4-form localized on the volume of the D5-brane. We then see that the amount of F_3 flux along the cycle $\tilde{\Sigma}$ dual to Σ changes by k units in the D3-number violating process.

$$\Delta \int_{\tilde{\Sigma}} F_3 = \int_{t=t_0-\delta t} \Delta \int_{\tilde{\Sigma}} F_3 - \int_{t=t_0+\delta t} \Delta \int_{\tilde{\Sigma}} F_3 = \int_{\Pi_4} dF_3 = \int_{\Pi_4} \delta(D5) = 1 \tag{B.3}$$

where Π_4 is Σ times the interval $[t_0 - \delta t, t_0 + \delta t]$. This implies that the contribution to the C_4 tadpole from $N_{\rm flux}$ changes by k units

$$\Delta \int_{X_6} H_3 F_3 = \int_{X_6} H_3 \Delta F_3 = \int_{\Sigma} H_3 \int_{\tilde{\Sigma}} F_3 = k \tag{B.4}$$

therefore compensates the disappearance of k D3-branes.

Some remarks are in order

- The above description suggests that NS5-brane instantons can mediate the decay of D3-branes in the presence of RR 3-form field strength flux F_3. This is an additional modification not taken into account in the usual K-theory groups discussed in the literature, due to the NS nature of the instantonic brane. Certainly the classification of D-brane states in the presence of general fluxes is still an open question and deserves further study.

- RR tadpole cancellation works as usual, taking into account the charges carried by the fluxes. This also implies that instanton processes mediated by euclidean D-branes in the presence of H_{NS} do violate D-brane number, but conserve the charges under the RR fields.

- We conclude by pointing out that the above instantons can be regarded as domain walls in time. In fact, some of them are supersymmetry preserving, and should connect different supersymmetric configurations of fluxes and D-branes. In fact, using one of the above spatial coordinates as time, our above instanton processes become spatial domain walls, separating regions of space with different fluxes and D-branes, of the kind studied in [47].

- In conclusion the above instantons in our models simply represent possible tunneling processes connection our supersymmetric configurations with other supersymmetric configurations. Hence the instanton processes, and more generally the possibility of D-brane disappearance due to the fluxes, can be safely ignored in our context. (It would be an open issue in non-supersymmetric models, where an analysis of the energetics of the barrier would be requrired to ensure enough metastability).

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